Monetary Policy Normalization in the New Normal: The Role of Quantitative Tightening

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Abstract

Do the pace, timing and announcement of balance sheet unwinding matter? This paper investigates the implications of various Quantitative Tightening strategies, examining both implementation and announcement effects. We focus on the consequences for financial stability and real variables, particularly the impact of reintroducing government bonds to the market, the role of reserves demand and balance sheet costs of financial intermediaries during the unwinding process. We also explore the dynamics associated with announcement effects. We present empirical evidence on the effects of QT on financial variables and develop a quantitative model with a banking sector to understand the dynamics of different QT strategies. We explore optimality and compare cases of commitment and discretion, as well as credibility and limited commitment. Our findings indicate that announcing QT with sufficient anticipation yields better macro-financial outcomes. While sales initially have a relatively short-term stimulative effect due to agents precautionary motives, negative implementation effects eventually arise. Announcing passive unwinding followed by conducting sales leads to lower welfare and higher output volatility. Optimal QE is aggressive and optimal QT is gradual. QT should be more gradual when the maturity of debt is higher and reserves demand is higher. Under the optimal dual policy with commitment, the output gap closes and fully stabilizes 12 quarters earlier than observed in the data.

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1 Introduction and Motivation

Following the Great Recession, major central banks adopted monetary policies characterized by near-zero interest rates and significant balance sheet expansions, known as Quantitative Easing (QE). As economic activity recovered and inflation began to rise, central banks sought to strike an optimal balance between raising interest rates and reducing balance sheets in order to mitigate inflationary pressures, regain policy space, and minimize adverse effects on financial markets and the broader economy.

While QE has been extensively studied in the academic literature following the Great Financial Crisis, the normalization process—Quantitative Tightening (QT)—remains less explored. As balance sheets are expected to complement short-term interest rates in future monetary policy frameworks, understanding the unwinding process, including state-dependent asymmetries, timing and pace implications, and interactions with conventional monetary policy, is crucial for effective policy design.

The motivation for this paper is to understand the economic outcomes, particularly the evolution of macro-financial variables, under various QT strategies. Both the pace of QT and the size of the balance sheet play critical roles in shaping these dynamics. The modeling of paces will be based on the strategies employed by major central banks, including active asset sales, fully passive unwinding, partial reinvestment of maturing assets, and full, indefinite reinvestment of maturing assets. Specifically, we ask whether the timing and pace of the exit matter, and how the economy responds to anticipated versus unexpected exits. QT outcomes will be shown to be highly dependent on factors such as the liquidity services of reserves, equity/net-worth costs, and the demand and maturity structure of government bonds.

This paper makes three main contributions. First, we compare the outcomes of different QT strategies and analyze potential trade-offs between various paces, ranging from one-time asset sales to passive unwinding. These strategies are evaluated based on discounted output volatility and welfare. Second, we examine the effects of policy announcements, focusing on the timing of such announcements and their consequences, given the same policy and the model's underlying frictions, such as reserve liquidity benefits and balance sheet costs. Finally, we explore optimal policy using a constrained optimal projection approach, which is a form of sufficient-statistics-based counterfactual analysis, considering both commitment and discretion scenarios. This allows us to address key questions surrounding balance sheet unwinding strategies: Should interest rate lift-off occur before or after the start of QT? Should QT be gradual or as aggressive as the QE conducted by most central banks? Outside the ZLB, how do QT and interest rate hikes complement each other? Additionally, we investigate the role of limited commitment as an intermediate case between full commitment and time consistency, as well as the implications of inattention and credibility.

We show that the negative effects of QT are mitigated when the policy is announced with greater anticipation. The higher the liquidity benefits and the greater the leverage costs for banks, the more anticipation is required. We solve the model non-linearly, simulating a crisis that drives the economy to the Zero Lower Bound, after which the central bank conducts QE and proceeds with different exit strategies. The worst-performing strategies, in terms of discounted output volatility and welfare, are unexpected aggressive sales. While sales provide short-term stimulation due to precautionary motives—as agents expect tighter conditions in the near future—passive unwinding leads to smoother implementation effects.

Regarding optimal policy, a planner with a commitment device can be more aggressive than one operating under discretion, as the former can leverage the power of forward guidance. Under discretion we observe an asymmetry in the unconventional monetary policy pace: QE is implemented aggressively as the planner seeks to capture the benefits of relaxing banks' constraints and increasing capital gains. However, the unwinding process is slower and more cautious due to the tightening effect from the reduction of reserves and capital losses. When the debt maturity is longer, shocks are more amplified, leading to a slower optimal QT pace, similar to scenarios where reserves provide higher liquidity benefits. In additional exercises, we show the for lower degree of agent attention , finite planning, and lower credibility, the QT pace should be lower and the use of balance sheet higher.

The asymmetries between QE and QT arise from several sources. First, there is state-dependency: QE is conducted at the ZLB, while QT occurs outside of it, once inflation is converging to its target and the output gap is nearing zero. Second, there is asymmetry in the optimal pace, as discussed in the previous paragraph. Lastly, the liquidity role of reserves is captured by a non-linear function related to the tightness of banks' constraints. When financial intermediaries face a scarcity of reserves, the tightness increases exponentially, which raises leverage and reduces the available credit space.

Finally, we highlight the importance of an exit strategy and the risks associated with a permanent reinvestment strategy. In such a scenario, banks' net worth is adversely affected by lower term and credit spreads, which increases leverage and reduces the available credit space for providing loans to firms, ultimately harming investment.

Background

In the United States, the Federal Reserve conducted its first QT from October 2017 to September 2019, with a second round starting in May 2022. QT is uncharted territory for most central banks due to the lack of experience with balance sheet reduction outcomes and potential asymmetries compared to Quantitative Easing (QE). Understanding QT is crucial in the ”new normal” monetary policy implementation, where balancing conventional interest rate adjustments and balance sheet management has become a key challenge. Looking forward, as central banks navigate the new framework with ample reserves system of monetary policy, the interplay between conventional tools and balance sheet management will be a defining feature of their strategies to ensure economic stability. When designing the QT policies, the central banks should avoid large swings in government bond prices and creating any reserve scarcity that can lead to market stress episodes. For instance, in September 2019, the US repo market tanked as the Fed was engaged in an episode of QT. Similarly, in March 2020 there was an episode of bank illiquidity at the outset of the Covid-19 epidemic. and also in the British market turmoil during October 2022.

QT strategies have varied across central banks. In May 2022, the Federal Reserve announced plans to reduce its balance sheet size by adjusting reinvestments of principal payments from securities held in the System Open Market Account (SOMA). The strategy involves gradually increasing reinvestment caps, allowing the balance sheet to shrink predictably, and stopping the decline when reserve balances are deemed ample. For instance, the European Central Bank (ECB) has taken a more gradual approach compared to others, both in terms of policy rate normalization and QT timing and pace. In February 2023, the Riksbank accelerated its asset reduction by actively selling government bonds to stabilize inflation. The Bank of Canada halted QE in October 2021 and began QT in April 2022 by ceasing government bond purchases and reinvestments. The Reserve Bank of Australia ended its bond purchase program in February 2022 and pursued passive QT from May 2022 by not reinvesting maturing bonds, reducing its bond holdings by an estimated AUD 4 billion in 2022 and AUD 13 billion in 2023.

QT can be implemented via different methods. One is active sales: the pace is controlled by the central bank, but entails financial stability risks. The second option is a full passive unwinding, where the central bank does not reinvest the proceeds from maturing assets and lets the balance sheet decline passively. The reduction pace is more irregular than the former method, as it depends on the coupon redemptions. The third way is to conduct a partial reinvestment, where a cap is imposed to the reduction of the balance sheet. Finally, the slowest strategy is a full and indefinite reinvestment of maturing assets.

Quantitative Tightening has potential benefits but also entails risks. The reasons to conduct QT are to regain policy room for future interventions, to mitigate negative effects in the financial markets of a large balance sheet (footprint/lack of collateral) and to withdraw policy accommodation to support monetary policy stance. The reduction of the risks of fiscal dominance has also been cited as one of the most important factors to take into account while conducting QT, as Schnabel (2023) noted. But also there are several risks: the decline in reserves can lead to financial instability episodes. The financial system structurally needs much more reserves than before the financial crisis, because liquidity requirements are higher and because QE caused higher liquid liabilities issuance, according to Acharya et al. (2023). This is known as the QE "ratchet effect". To mitigate these risks central banks as the FED and Bank of England introduced liquidity facilities once they started QT: the Standing Repo Facility and the Short Term Repo respectively.

Table 1 shows the evolution of the Balance Sheet sizes of the Federal Reserve, Bank of England, ECB and the Bank of Canada.

Table 1: Evolution of Size of BS/GDP

CB	Dec 2007	Dec 2014	Dec 2019	Dec 2021	Sept.2023
FED	6 %	25 %	19 %	37 %	30 %
BoE	6 %	21 %	29 %	47 %	38 %
ECB	16 %	21 %	36 %	69 %	51 %
BoC	3.5 %	4.7 %	5 %	19 %	11.5 %

Figure 1 shows the evolution of the holdings of the Federal Reserve Treasuries as a percentage of the total issuance.

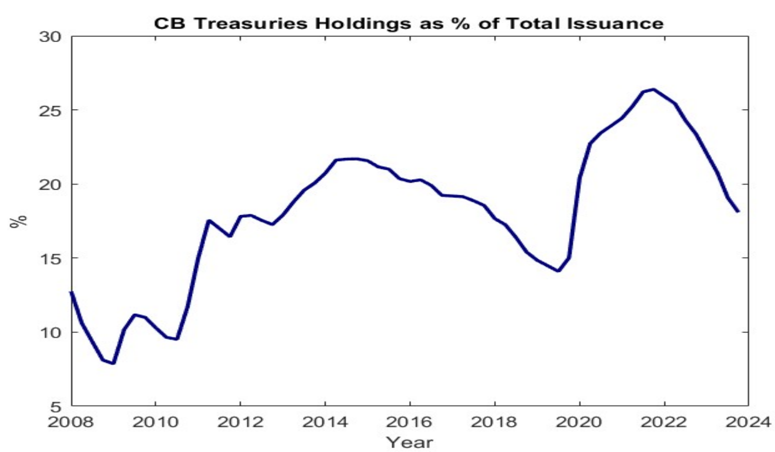

Figure 1: Treasury Holdings

Figure 2 shows the projected evolution of the decrease in the stock of government bonds held by the Federal reserve. After the first quarter of 2024 we project a prospective path of Treasuries and MBS as shares of nominal GDP computed on the assumption that Treasuries fall by $60 billion per month until the total balance sheet hits 18 percent of GDP and that MBS fall by $22 billion per month for the remainder of the projection period. Similar assumptions are conducted by Wright (2022). Figure 3 shows the evolution of the composition of the FED balance sheet size, as well as the increase after the pandemic of the ONRRP.

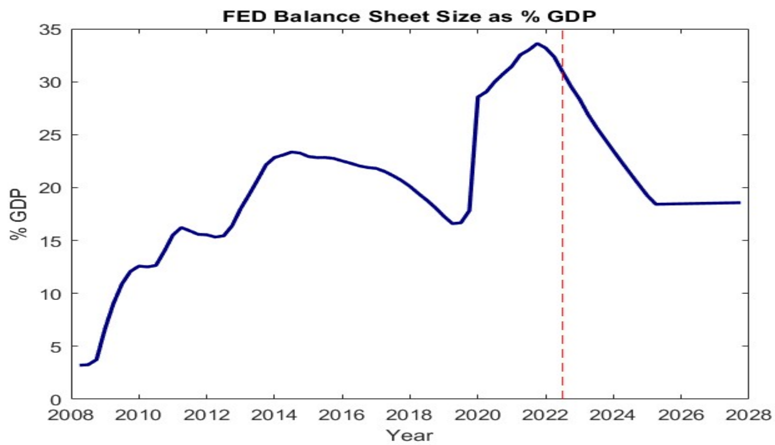

Figure 2: Balance Sheet over GDP

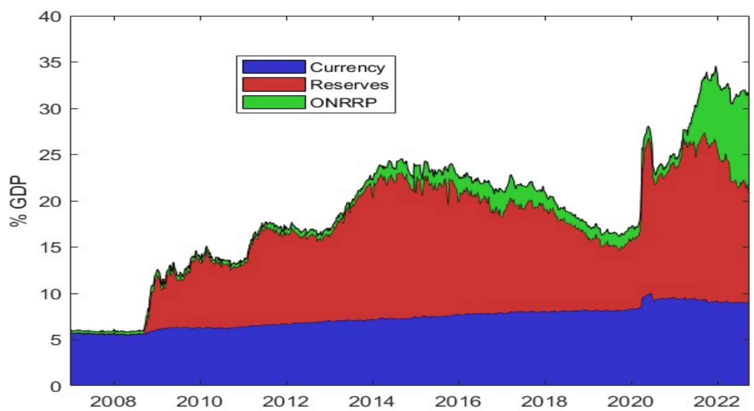

Figure 3: Balance Sheet Composition Evolution

Literature Review

This paper sits at the intersection of several strands of literature: studies on the management of the central bank's balance sheet as an active tool of monetary policy, research focused on monetary policy normalization, analyses of the departure from Wallace's Open Market Operations Neutrality, the segmented markets literature where financial intermediaries play roles in liquidity and credit management, and, finally, the literature on optimal monetary policy at the ZLB.

Benigno and Nistico (2020) analyze the effects on the dynamics of inflation and output under some different scenarios for fiscal and monetary policy interactions, and when the potential central bank losses might make an open market operation non-neutral. The neutrality depends on the degree of fiscal support that the Treasury offers to the monetary authority.

Benigno and Benigno (2022) extend the standard New Keynesian to focus on the optimal combination of reserves and interest rate policy. They distinguish between a classical Neo-Wicksellian framework where there is full satiation of liquidity and therefore reserves become effective, and an scenario where there is room for active reserve management and in which the interest rate on deposits is a combination of the natural interest rate and the interest rate on reserves. Elenev et al. (2021) study the fiscal sustainability in a NK model with financial intermediaries. They find that temporary unconventional monetary policy increases fiscal space and stimulates the economy. A permanent policy of this kind crowds out investment.

Boehl, Gavin, and Strobel (2022) study a similar model with banks as in Gertler and Karadi (2013) and they estimate the model for the US: they find evidence for a portfolio-rebalancing channel but also a novel feature: QE might be deflationary if the increase in supply due to higher credit and lower marginal costs exceeds the increase in the aggregate demand.

In Bhattarai, Eggertson, and Gafarov (2023), QE, from a non-commitment optimal policy problem point of view, works creating expectations of a future monetary expansion in a time-consistent equilibrium.

Bianchi and Bigio (2022) develop a model with banks' liquidity management with an OTC interbank market. They study how monetary policy can affect the mix between lending and liquidity risk exposure. Arce et al. (2020) compare two monetary policy regimes in a New Keynesian structure: a floor and a corridor systems.

The literature of QT structural models is quite limited so far. Airaudo (2023) explores a DSGE model where a regime switching between fiscal dominance and monetary dominance is allowed. Cantore and Meichtry (2023) develop a NK model with two types of households. They also explore the effects of the balance sheet unwinding, but from a pure household portfolio perspective. They find that, when close to the lower bound, raising the nominal interest rate prior to unwinding quantitative easing minimises the economic costs of monetary policy normalisation. The results imply that household heterogeneity in combination with state dependency amplifies the asymmetry. Foerster (2015) finds that selling off assets quickly produces a double-dip recession while slowly unwinding generates a smooth recovery. Expectations about the exit strategy influence the initial effectiveness of purchases. He uses a Markov-switching model based in a Gertler and Karadi (2011) framework. He studies QE as a credit intervention rather than a long-term government bonds purchases. In a very similar model structure, Karadi and Nakov (2021) conclude that exit should be gradual when banks recapitalization is costly. Harrison (2024) studies optimal policy in a NK model with portfolio frictions between long and short term bonds. Optimal QT is more gradual than QE due to this friction. Dominguez and Gomis-Porqueras (2022) also study monetary policy normalization but in a Lagos-Wright (2005) framework. With limited commitment in the assets markets and when government bonds can be used as collateral and exhibit liquidity premia among different maturities, then changes in the central bank balance sheet have implications for inflation and real allocations. They conclude that the normalization process should respond to the total debt issued in the economy relative to its target. Finally, Jiang (2023) show in a preferred-habitat model that when the central bank unwinds its bond purchase, slow adjustment by long-term investors requires liquidity traders/ short-term investors to absorb the imbalance, who demand a higher risk premium that creates excessive bond price decline and volatility in the short run. From the empirical literature point of view, Lee Smith and Valcarcel (2023) offer a granular analysis for the first QT experience in the US. They find evidence of tightening effects for financial variables and long-term spreads and lack of evidence of announcement effects. Lu and Valcarcel (2023) focus on the second QT: they find substantial announcement effects and stronger market response upon implementation. Lopez Salido and Vissing Jorgensen (2023) estimate the optimal reserve supply such that there's no liquidity shortage that induces a financial instability scenario. Acharya et al. (2023) show that during QE commercial banks financed reserve holdings with deposits and also issued lines of credit. During QT, there was no shrinkage of these claims on liquidity and this left the banking sector more sensitive to liquidity shocks that during 2019 and 2020 suffered the most. D'amico (2023) show that Treasury yield sensitivities to quantitative tightening supply surprises are on average larger than sensitivities to quantitative easing surprises. Finally, Du (2024) conduct an international analysis of the QT experiments: they conclude that QT announcements increase government bond yields, steepening the yield curve and signal a greater commitment to raising policy interest rates, but have more limited effects on most other financial market indicators. Active QT has a larger impact than passive QT, particularly on longer maturities.

2 Empirical Evidence

2.1 Implementation Effects

We start showing the liquidity effect strength comparing the QE episodes against the full reinvestment phase and the first US QT, from October 2014 until September 2019. We use a reduced-form regression to explore the relationship between the natural log of reserves and the liquidity cost as measured by the difference between the effective federal funds rate and the interest rate on reserves, following Lee Smith and Valcarcel (2023) and Acharya et al. (2023):

$$EFFR_t - IOR_t = \alpha + \beta \log(RES_t) + \epsilon_t$$

As observed in Figure 4 and Figure 5, there is an asymmetric strength of the liquidity effects between QE and QT. In the latter the effect is much more pronounced.

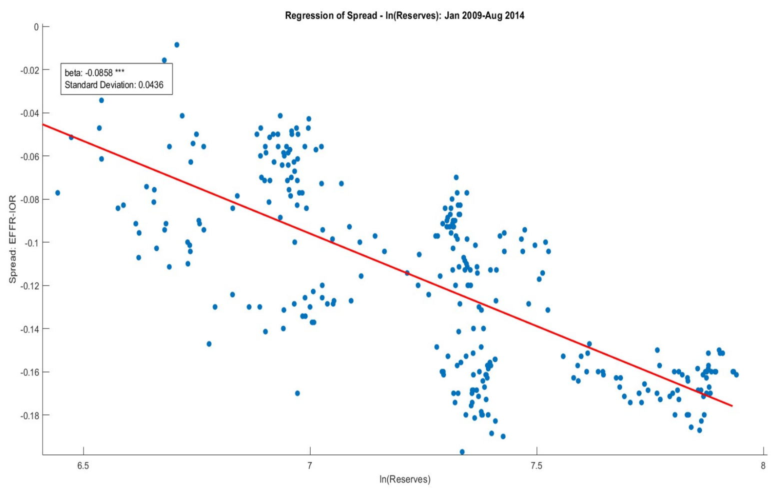

Figure 4: Jan 2009- Sept 2014

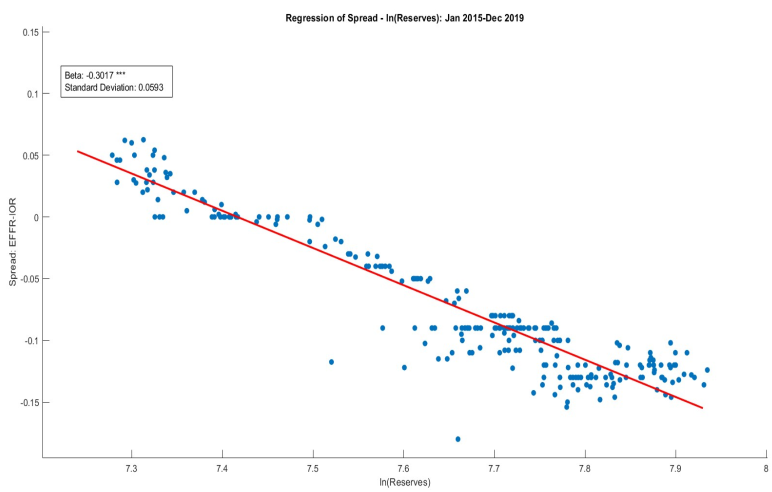

Figure 5: October 2014- September 2019

In Table 2 we regress the financial intermediaries reserves over total assets on the SOFR-IOR spread since the beginning of 2008.

Table 2: Regression Results: 2008 Q1 to 2023 Q4

Variable	**Coefficient**
Reserves over Total Assets	-0.97^{***}
	(0.13)

Now we proceed to use the same method to identify a contractionary reserves shock of Lee Smith and Valcarcel (2023) but focusing on the main categories of the Chicago Federal Reserve Financial Conditions Index: leverage and credit. Also, we will study the impact of a decrease in the reserve supply in the spread between the effective funds rate and the interest rate on reserves, the BAA Corporate Bond Yield relative to yield on 10-Year Treasury Constant Maturity , the 10 year government bond yield and the KBW NASDAQ bank index.[1]

To identify the decrease of reserves coming actually from QT Lee Smith and Valcarcel (2023) use two institutional facts of the Federal Reserve reserve supply structure and the auction of bonds.

Firstly, during both quantitative tightening (QT) episodes (2017-2019 and the one that began in May 2022), the Federal Reserve explicitly stated that it did not adjust reserve balances on a week-to-week basis in response to changes in money market or broader financial conditions, rendering the short-term supply curve perfectly inelastic. Secondly, weekly fluctuations in reserves could stem from changes in the Treasury Bill supply. However, these auctions are announced on Tuesdays, with winning bidders settling with the Treasury up to one week post-announcement. Consequently, any movement in spreads due to an announced increase in the bill supply should not affect the end-of-day reserve balances, which are announced on Wednesdays.

The VAR structure is: $x_t = \Big[100{*}\log(RES_t); (EFFR_t{-}IOR_t); (BAA_t{-}R10_t); (R10_t); (F_t); (100{*}\log(SOMA_t))\Big]$ where F is a rotating variable: credit, leverage, the KBW index.

During QT I, the average value of the KBW Banking Index was 102, with a standard deviation of 6.9. In QT II, these values were 95 and 14, respectively. The mean of the credit index was 0.074 during QT I and -0.13 during QT II, with a standard deviation of 0.05 across both periods. For the leverage index, the mean was -0.02 in QT I and 0.12 in QT II, while the standard deviations were 0.19 and 0.28, respectively.

Figure 6 shows the response of a negative 1 standard deviation to reserves on the list of variables. The time unit is weekly. The decrease of reserves lead to increases in the short term funding spread, the long-term bond rate, and we observe a decrease in credit and

[1]Here we use the 10 yr. Treasury yield.

increase in leverage. Also, a tightening effect can be visualized trough the banking KBW index.

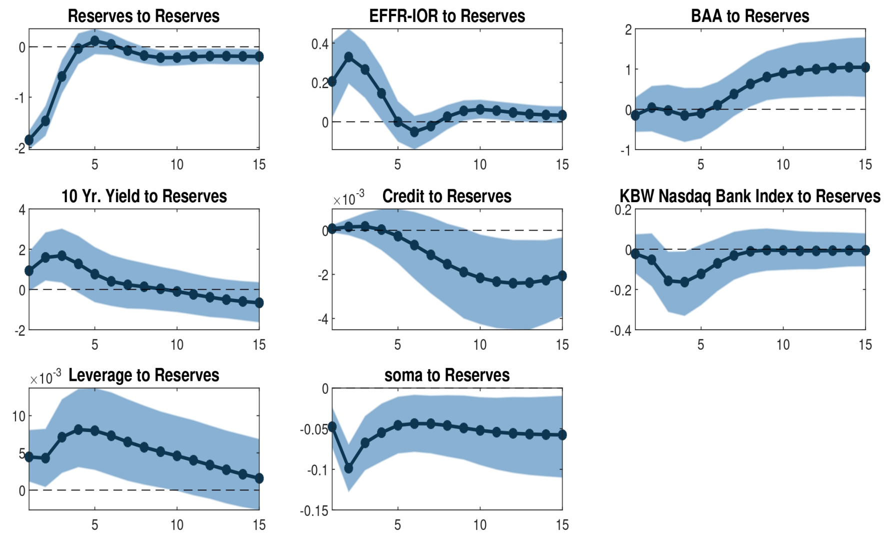

Figure 6: Oct 2017- Sept 2019. (Spreads wrt. IOR)

The results from the analysis for the QT II period are shown in the following figure. The responses are similar than in the first rounf of the US quantitative tightening, except for credit, that shows an initial increase. [2]

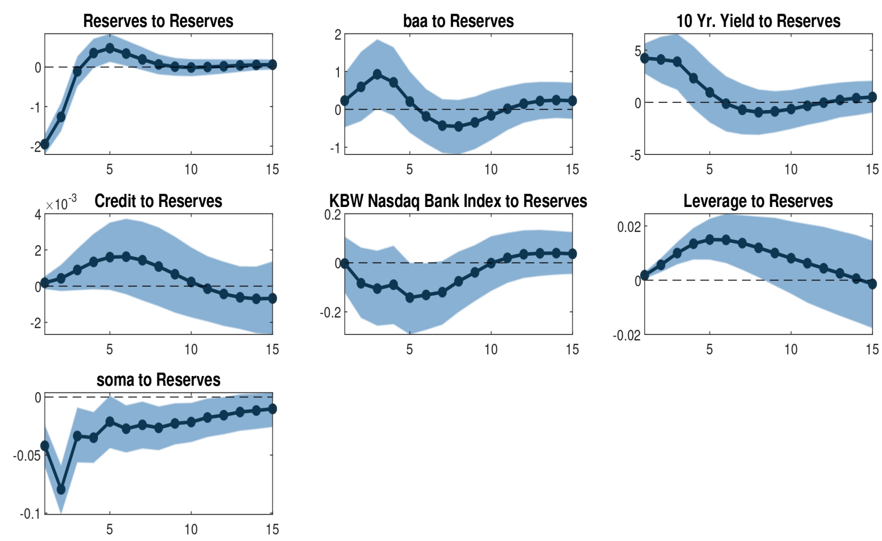

Figure 7: May 2022- Feb 2024. (Spreads wrt. IOR)

[2] As the Federal Reserve maintained a -7bp difference between the EFFR and the IOR during this period, we leave this spread out of the analysis.

2.2 Announcements

Now we proceed to study the announcement effects of QT, following Lee Smith and Valcarcel (2023), Lu and Valcarcel (2023) and Vissing-Jorgensen and Krishnamurthy (2011) . We study the two day change in the yields of selected treasury bonds and the term premium as calculated by Kim and Wright (2005), using three-factor arbitrage free nominal term structure model, of all the tapering, passive unwinding/reinvestment and the two QT scenarios. Tapering and QT2 are found to have significant announcement effects, specially on the 5 and 10 year treasury yields and the ten year term premium. To take into account the effects of announcements related to changes in the Fed Funds rates (conventional monetary policy) we control for the 30 days Fed Funds futures.

Table 3: Quantitative Tightening Announcement Effects on Treasury Yields

Date	Announcement	2-yr.	5-yr.	10-yr.	30-yr.	TP10
May 22, 2013	Taper	0.00	0.07	0.08	0.06	0.05
Jun 19, 2013	Taper	0.6	0.24	0.21	0.15	0.13
Dec 18, 2013	Taper	0.01	0.11	0.9	0.3	0.05
May 21, 2014	Unwind/FR	0.02	0.04	0.04	0.05	0.02
Jul 9, 2014	Unwind/FR	-0.05	- 0.04	-0.03	0.00	-0.03
Jul 17, 2014	Unwind/FR	0.01	-0.02	-0.05	-0.06	-0.03
Aug 20, 2014	Unwind/FR	0.03	0.05	0.01	-0.02	0.02
Sep 17, 2014	Unwind/FR	0.04	0.07	0.03	0.00	0.03
Oct 29, 2014	Unwind/FR	0.06	0.05	0.02	-0.02	0.03
Jan 12, 2017	Unwind/FR	0.01	0.01	0.02	0.03	0.00
Apr 5, 2017	Unwind/FR	-0.01	-0.01	-0.02	0.00	-0.02
May 24, 2017	Unwind/FR	-0.01	-0.06	-0.04	-0.03	-0.02
Jun 14, 2017	QT1	-0.03	-0.03	-0.05	-0.09	-0.03
Sep 20, 2017	QT1	0.05	0.05	0.03	-0.01	0.03
Jan 26, 2022	QT2	0.16	0.1	0.03	-0.03	0.03
Mar 16, 2022	QT2	0.09	0.07	0.05	0.01	0.05
May 4, 2022	QT2	-0.07	0.00	0.08	0.12	0.03
Sep 21, 2022	QT2	0.15	0.16	0.13	0.06	0.07
Nov 2, 2022	QT2	0.17	0.09	0.07	0.04	0.02

Table 4: Quantitative Tightening Announcement Effects on Treasury Yields

Balance Sheet Policy Event	2-yr.	5-yr.	10-yr.	30-yr	TP10
All Events	0.72*	0.99**	0.73*	0.3	0.45*
	(0.31)	(0.38)	(0.38)	(0.36)	(0.2)
Only Taper Events	0.06	0.42**	0.38**	0.24*	0.23***
	(0.11)	(0.14)	(0.14)	(0.13)	(0.07)
Only FR Events	0.09	0.08	-0.02	-0.05	0.00
	(0.2)	(0.25)	(0.25)	(0.23)	(0.13)
Only QT1	0.01	0.01	-0.02	-0.1	0.00
	(0.09)	(0.11)	(0.11)	(0.11)	(0.06)
Only QT2	0.5**	0.42**	0.36**	0.2	0.2**
	(0.15)	(0.18)	(0.18)	(0.17)	(0.1)

[a] Coefficients β_{QT} from the regression: $\Delta y_n^t = \beta_{QT} QT_t + \alpha X_t + \varepsilon_t$, where Δy_n^t is the two-day change in the n-year constant maturity Treasury yield or term premium and QT_t is a dummy variable, which takes a value of 1/t on the dates listed above. X_t is the 30 days Fed Funds Futures. Units in pp.

Notes: OLS standard errors are reported in parentheses. Sample Period: January 2008–December 2023. $p < 0.01$, $p < 0.05$, $p < 0.10$.

3 The Quantitative Model

The model consists on a New-Keynesian model where households can invest in deposits or long-term bonds, subject to portfolio adjustment costs and firms finance their capital investments with bank loans. Banks are modeled as in Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler and Karadi (2013), henceforth GKK. Our setting extends the GKK banks with reserve that feature liquidity benefits and loans/net-worth adjustment costs. The government bonds also have a liquidity premium over loans. The Central Bank sets the interest rate on reserves and controls the balance sheet. The process of the balance sheet will be augmented to take into account announcement effects.

3.1 Households

The households that can invest in deposits and in long-term government bonds, subject to portfolio costs.

$$V_t = \max_{\{C_t, L_t, B_t^H, D_t\}} \left[\frac{(C_t)^{1-\phi}}{1-\phi} - \frac{\chi}{1+\psi} L_t^{1+\psi} \right] + \beta \, \mathbb{E}_t \, V_{t+1}$$

$$st.$$

$$C_t + D_t + T_t + Q_t^B [B_t^H + \Phi_t^B] = W_t L_t + R_t^D D_{t-1} + B_{t-1}^H R_t^B Q_{t-1}^B + \sum_{B,F} \Pi_t$$

Where D_t are deposits and B_t^H are long term bonds. They will pay taxes T_t and receive profits from banks and firms, Π_t

The portfolio adjustment costs are given by $\Phi_t^B = \frac{1}{2} \frac{\kappa^H (B_t^H - \overline{B}^H)^2}{B_t^H}$. They capture that households pay a cost when the bonds holdings exceed the frictionless capacity level, $\overline{B}^H$.

Demand for long-term bonds above its frictionless capacity level is increasing in the excess return with an elasticity of the inverse of the curvature parameter κ^H.

$$B_t^H = \overline{B}^H + \frac{\mathbb{E}_t \, \Lambda_{t+1} (R_{t+1}^B - R_t^d)}{\kappa^H} \tag{1}$$

The return of the long-term government bonds is given by:

$$R_{t+1}^B = \frac{c + \gamma^b Q_{t+1}^B}{Q_t^B} \tag{2}$$

We will normalize the coupon to 1. γ^b is the Woodford decaying coupon, that will be used to match the US debt average maturity. The price of the bond is denoted Q_t^B.

3.2 Firms

The production side is characterized as in a standard New-Keynesian model, where the final goods producers compose intermediate goods varieties and intermediate goods producers are subject to quadratic adjustment costs for adjusting prices. Their technology is a Cobb-Douglas production function. There are also capital producers subject to adjustment costs.

3.2.1 Final Goods Producers

The final goods sector operating under perfect competition:

$$\max_{Y_t(i)} \Pi_t = P_t Y_t - \int_0^1 P_t(i) Y_t(i) dj$$

$$st.$$

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

Their maximization problem imply a demand function:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t$$

3.2.2 Intermediate Goods Producers

Their production function is:

$$Y_t(i) = Z_t(\xi_t K_{t-1}(i))^{\alpha} (L_t(i))^{1-\alpha}$$

Z_t is an aggregate TFP shock and ξ_t is a capital quality shock as in Merton (1973). Both follow an AR(1) process.

These firms are subject to quadratic adjustment costs a la Rotemberg: $AC_t = \frac{\phi^P}{2} \left(\frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \right)^2 y_t$. They maximize the stream of current and future dividends.

The marginal costs are given by:

$$mc_t = \left(\frac{1}{\alpha} \right)^{\alpha} \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \frac{1}{z} w_t^{1-\alpha} (R_t^k)^{\alpha}$$

The solution from optimal price setting delivers a New Keynesian Phillips Curve:

$$1 - \epsilon + \epsilon mc_t = \phi^P (\pi_t - \pi) \pi_t - \phi^P \, \mathbb{E}_t \left[\Lambda_{t,t+1} \frac{y_{t+1}}{y_t} \pi_{t+1}^2 (\pi_{t+1} - \pi) \right] \qquad (3)$$

The firms finance their capital expenditures with a bank loan. [3]The return on the loan by the bank is given by:

[3]Sims and Wu (2021) study an environment where wholesalers finance a fraction of the investment issuing long term bonds. Their QE mechanism is similar than in our setting, but smoother.

$$R^F_{t+1} = \frac{R^K_{t+1} + (1-\delta)\xi_{t+1}Q^K_{t+1}}{Q^K_t} \tag{4}$$

where R^K_{t+1} is the net period income flow to the bank from a loan.

The capital stock will follow:

$$K_{t+1} = \left[I_t + (1-\delta)\xi_t K_t\right]$$

3.2.3 Capital Goods Producers

They choose investment on capital subject to quadratic adjustment costs.

$$\max_{I_t} \mathbb{E}_t \sum_{j=t}^{\infty} \Lambda_{t,t+1}\left[Q^K_t I_t - \left[1 + s(\frac{I_j}{I_{j-1}})I_j\right]\right]$$

$$st.$$

$$s(\frac{I_j}{I_{j-1}}) = \frac{\phi_k}{2}(\frac{I_j}{I_{j-1}} - 1)^2 I_j$$

The resulting price for capital is:

$$Q^K_t = 1 + \phi_k \frac{I_j}{I_{j-1}}\left(\frac{I_j}{I_{j-1}} - 1\right) + \frac{\phi_k}{2}\left(\frac{I_j}{I_{j-1}} - 1\right)^2 - \phi_k \Lambda_{t,j}\frac{I^2_{j+1}}{I^2_j}\left(\frac{I_j}{I_{j-1}} - 1\right) \tag{5}$$

3.3 Banks

There is a continuum of banks that collect deposits from the households, and with their own net worth invest in claims on capital stock, in reserves and in government bonds.

Banks maximize the expected discounted value of their net-worth. The value of each bank is given by:

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty}(1-\sigma)\sigma^{j-1}\Lambda_{t,t+1}n_{j,t+1}$$

where $1-\sigma$ is the exit probability and n is the net worth.[4] When bankers exit, they give retained earnings to the household. An equal number of bankers enter at the same time.

[4] This works as a dividend payout to the households

Their balance sheet is:

$$Q_t^K S_{j,t} + Q_t^B B_{j,t} + M_{j,t} = D_{j,t} + N_{j,t} \tag{6}$$

The financial intermediaries can divert an endogenous function $\Theta(M_t/N_t)$ of the assets and transfer that to the households. This creates an agency problem that imposes an incentive constraint as in Gertler and Kiyotaki (2010) in that the continuation value of bankers has to be greater than a fraction of the assets. If the banker diverts, depositors can force him to bankruptcy and then leave the financial sector.

$$V_{j,t} \geq \Theta\left(\frac{M_{j,t}}{N_{j,t}}\right)\left(Q_t^K S_{j,t} + \Delta^L Q_t^B B_{j,t}\right) \tag{7}$$

We assume that each asset has a different quality, or carries different weights when fulfilling SLR and LCR-type requirements. Government bonds are a better asset (harder to abscond with) than credit by a fraction Δ^L, but they are inferior to reserves, which have zero weight when computing leverage.

One main difference with the standard literature is that the tightness of the constraint is an endogenous function: it is decreasing in the reserves-net worth ratio. This way, reserves have liquidity benefits, and are an important factor for the pace of QT, as expressed by Lopez Salido and Vissing Jorgensen (2023). The functional form, based on Onofri, Peersman, and Smets (2023), is the following:[5] [6]

$$\Theta(M_t/N_t) = \frac{1}{\exp\left(\theta(1+\gamma(\frac{M_{j,t}}{N_{j,t}}))\right)} \tag{8}$$

The convex functional form and calibration capture that the function is relatively flat around high levels of reserves over net-worth and gets steeper in an exponential way as the ratio falls toward low values. In Appendix 9.9 we study a partial equilibrium model for banks that demand reserves because they decrease costs when there is a deposit withdrawal shock. Government bonds can also be used, but are more illiquid. This behaves similar to a LCR requirement. The endogenous tightness of the constraint in this model can reflect those features: liquidity benefits from reserves and a short-term rate difference. Finally, we introduce a cost of deviating from a target of loan-to-net worth ratio. The bank pays a quadratic cost. This is introduced to capture the idea of slow-moving capital and that even

[5] We will try with another functional form, as in Akinci et al. (2023). This functional form will be relatively flat around high levels of reserves/net worth and will be steeper when this ratios is low. The current one shows a smoother process. The alternative is: $\Theta(M_t/N_t) = \theta(1 - \frac{\lambda}{\kappa}(\frac{M_t}{P_t})^{\kappa})$

[6] Cecchetti and Schoeholtz (2019) claim that the FED's LCR appears to place more weight on reserves than in other HQLA as government bonds.

with policy announcements, a bank equity buffer is important and then the lack of it can create effects that lasts longer. Banks can not simply obtain their optimal level of capital in a frictionless way. A similar idea is captured in contributions that study the impact of negative interest rates or reversal rates, as Ulate (2022) , Eggertson et al. (2023) and Abadi, Brunnermeier, and Koby (2023). Miao, Shen, and Su (2023) use a similar cost to prevent frictionless arbitrage between firm equity and long-term government bonds position by banks. Ulate (2022) motivates this cost as banks are punished if their leverage is too low. In appendix 9.5, we provide a microfoundation of the equity decision of the banks in a similar way as Karadi and Nakov (2021) and Akinci and Queralto (2022).

The cost is homogeneous of degree one, decreasing in net worth and increasing in loans. We assume that the marginal cost is a convex function:

$$\frac{dC(N_t, S_t)}{dS_t} = \kappa^L \left(\max\left\{ \frac{Q_t^K S_t}{N_t} - \frac{S}{N}, 0 \right\} \right)^2 \tag{9}$$

With both the adjustment costs in net-worth and the liquidity benefits of reserves, we capture the idea that even when QT announcements are made well in advance, they can have more significant effects throughout time when there is slow moving capital, and banks do need reserves, as emphasized by Lee Smith and Valcarcel (2023).

The evolution of the banks net-worth can be characterized as:

$$N_{j,t} = (R_t^F - R_{t-1}^D) Q_{t-1}^K S_{j,t-1} + (R_t^B - R_{t-1}^D) Q_{t-1}^B B_{j,t-1} + (R_{t-1}^M - R_{t-1}^D) M_{j,t-1}$$
$$+ R_{t-1}^D N_{j,t-1} - C(N_{j,t-1}, Q_{t-1}^K S_{j,t-1})$$

The net worth can be solved forward as:

$$N_{j,t} = \mathbb{E}_t \sum_{s=1}^{\infty} \left\{ \left[C_{j,t+s} - (R_{j,t+s}^F - R_{j,t+s}^D) Q_{j,t+s}^K S_{j,t+s} - (R_{j,t+s}^B - R_{j,t+s}^D) Q_{j,t+s}^B B_{j,t+s} - \right.\right.$$
$$\left.\left. (R_{j,t+s}^M - R_{j,t+s}^D) M_{j,t+s} \right] \prod_{l=1}^{s} \left(R_{j,t+l}^d \right)^{-1} \right\}$$

The interest-rate spreads from the optimality conditions are given by:

$$\mathbb{E}_t\,\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}\big(R_{t+1}^F - R_t^d - C_S'\big) = \frac{\lambda_t}{1+\lambda_t}\Theta(M_t/N_t) \tag{10}$$

$$\mathbb{E}_t\,\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}\big(R_{t+1}^B - R_t^d\big) = \frac{\lambda_t}{1+\lambda_t}\Delta^L\Theta(M_t/N_t) \tag{11}$$

$$\mathbb{E}_t\,\Lambda_{t,t+1}\Omega_{t+1}\Pi_{t+1}^{-1}\big(R_t^M - R_t^d\big) = \frac{\lambda_t}{1+\lambda_t}\frac{d\Theta(M_t/N_t)}{dM}(Q_t^K S_t + \Delta^L Q_t^B B_t) \tag{12}$$

where $\Omega_{t+1} = 1-\sigma+\sigma\frac{dV_{t+1}}{dN_{t+1}}$ is the bankers augmented stochastic discount factor.

As the banker's problem is linear, we can aggregate across intermediaries. Entering bankers receive an equity endowment X. The law of motion of aggregate net-worth is then:

$$N_t = \sigma\Bigg[(R_t^F - R_{t-1}^D)Q_{t-1}^K S_{t-1} + (R_t^B - R_{t-1}^D)Q_{t-1}^B B_{t-1} + (R_{t-1}^M - R_{t-1}^D)M_{t-1} + R_{t-1}^D N_{t-1} - C\Bigg(S_{t-1}, N_{t-1}\Bigg)\Bigg] + X \tag{13}$$

The aggregate adjusted leverage is:

$$\phi_t = \frac{Q_t^K S_t + \Delta^L Q_t^B B_t}{N_t} \tag{14}$$

After solving for the first order conditions of the bankers, and combining them, we arrive to an expression for the maximum adjusted leverage:

$$\bar{\phi}_t = \frac{\mathbb{E}_t\,\Lambda_{t,t+1}\Omega_{t,t+1}\Bigg(R_t^D - C_t\Bigg)}{\Theta_t(M_t/N_t) - \mathbb{E}_t\,\Lambda_{t,t+1}\Omega_{t,t+1}(R_{t+1}^F - R_t^D)\Bigg[1+\Psi_t\Bigg]}$$

where $\Psi_t = \left(\frac{M_t\Theta_t'(M_t/N_t)}{\Theta_t(M_t/N_t)}\right)$ are the reserves liquidity benefits. Details of the derivation are provided in appendix 9.4.

The maximum adjusted leverage depends positively on the funding costs (the interest rate on deposits) as depositors are more willing to invest in the bank increasing its value and negatively on the net-worth costs and the tightness of the constraint. Higher credit spreads and liquidity benefits increase the maximum leverage that is available, due to higher expected profitability.

Quantitative Easing/Open Market Operations are neutral in this framework when there are no convenience yields for reserves ($\Theta()$ is a scalar), and no spreads between capital and deposits. In this scenario we have a version of the Wallace's Neutrality. This happens when

$\lambda_t = 0$.

3.4 Government

3.4.1 Fiscal Authority

The Fiscal Authority finances government spending, and debt payments with taxes from households, central bank remittances and issuing long term debt.

Therefore, we can express the fiscal authority budget constraint as:

$$T_t + Q_t^B B_t^G + \Pi_t^{CB} = G_t + Q_{t-1}^B B_{t-1}^G R_t^B \tag{15}$$

In the baseline model, we will have a fixed government debt supply, B_t^G. In one of the policy coordination exercises, we will study debt supply management throughout a QT transition. Government spending follows an AR(1) process with fiscal stimulus/automatic stabilizers.

$$B_t^G = (1-\rho_{B^G})B^G + \rho_{B^G} B_{t-1}^G + \epsilon_t^{B^G} \tag{16}$$

$$G_t = (1-\rho_g)G + \rho_g G_{t-1} + \theta^G (Y - Y_t) + \epsilon_t^G \tag{17}$$

3.4.2 Central Bank

The central bank sets the reserves interest rate following a Taylor Rule, that is subject to an (occasionally) binding zero-lower bound constraint.[7]

$$R_t^M = \max\left\{ \left(R_{t-1}^M\right)^{\rho_i} \left[R^M \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \right]^{1-\rho_i} \epsilon_t^M, 1 \right\} \tag{18}$$

It will also control the size of the balance sheet, $M_t = Q_t^B B_t^{CB}$. In the Quantitative Tightening scenario, it will decrease the stock of reserves in the banking sector selling government bonds. The purchases/sales of government bonds by the central bank will be modeled as:

$$B_t^{CB} = \min\left\{ (1-\rho_{CB})B^{CB} + \rho_{CB} B_{t-1}^{CB} + \sum_{j\geq 0}^{T} \epsilon_{t|t-j}^{CB}, \overline{B}^{CBmax} \right\} \tag{19}$$

The last term is the path of QE/QT announcements at period t-j for $j \geq 0$ and realised in time t. $\overline{B}^{CBmax}$ is a maximum volume of government bonds that the central bank can

[7]For example, Ulate (2022) and Abadi, Brunnermeier, and Koby (2023) explore in a similar setting the consequences of setting an interest rate slightly below zero. We abstract from this analysis since in the US the Central Bank did not follow this policy.

purchase. In the US and the UK it is set at 0.7 of each of the different maturities issuance. We will analyze its consequences in section 5.5. In the appendix 9.17 we provide a rationale to modeling QE this way.

The profits to be rebated to the fiscal authority are given by:

$$\Pi_t^{CB} = B_{t-1}^{CB} Q_{t-1}^B R_t^B - Q_t^B B_t^{CB} + M_t - R_{t-1}^M M_{t-1}$$

Replacing by the balance sheet:

$$\Pi_t^{CB} = B_{t-1}^{CB} Q_{t-1}^B R_t^B - R_{t-1}^M M_{t-1}$$

The Central Bank conducts Quantitative Tightening via different paces. A scenario of active sales is one where the stock of debt in hands of the monetary authority is strictly decreasing. A fully passive unwinding process lets the balance sheet decrease at the decaying coupon factor. A partial reinvestment pace conduits a more conservative pace than the latter one. Finally, fully and indefinite reinvestment is one where the size of the balance sheet is kept fixed. The increase of output is the only way to decrease the ratio reserves/GDP in this last scenario. We summarize the paces, that are obtained with sequences of the QE/QT shocks explained above:

- Active Sales $B_t^{CB} < \gamma^b B_{t-1}^{CB}$
- Fully Passive Unwinding $B_t^{CB} = \gamma^b B_{t-1}^{CB}$
- Partial Reinvestment: $B_t^{CB} = (\gamma^b + \bar{\gamma}) B_{t-1}^{CB}$
- Full and Indefinite reinvestment: $\Delta(B_t^L) = 0$ so $B_t^{CB} = B_{t-1}^{CB} = B^{CB}$

Finally, we can consolidate the government budget constraint using the remittances to get:

$$\underbrace{G_t - T_t + B_t}_{\text{Fiscal Space}} = \underbrace{B_{t-1}^{PR}\left[c + \gamma^b Q_t^B\right]}_{\text{Debt Payments (held by private sector}} + \underbrace{R_{t-1}^M M_{t-1}}_{\text{Reserves as a Fiscal Burden}}$$

3.5 Market Clearing and Equilibrium Conditions

- $L_t = L_t^d$
- $S_t = K_t$

- $B_t^G = B_t^H + B_t + B_t^{CB}$
- $\Pi_t^F = Z_t Y_t - W_t L_t - R_t^K K_T$
- $Y_t = C_t + I_t(1 + s\big(\frac{I_t}{I_{t-1}}\big)) + G_t + \frac{\kappa^P}{2}(\Pi_t - \Pi)^2 Y_t$

A **perfect-foresight equilibrium** consists of sequences $\{C_t^j, L_t^j, B_t^H, D_t\}$, $\{L_t, K_t, I_t\}$, $\{M_t, D_t, B_t, S_t\}$, optimal prices and $\{W_t, Q_t^K, Q_t^B, T_t^U, T_t^{HTM}, B^{CB}, R_t^M\}$ such that:

- Decisions C_t^j, L_t^j, D_t, B_t^H solve the household's problem, taking wages, taxes/transfers as given and marginal utility Λ_t are consistent with the households solution;
- Decisions K_t, L_t solve intermediate firms problem taking wages as given;
- Decisions M_t, B_t, S_t solve banks problem taking the interest rate on reserves as given, and where aggregate net worth follows the law of motion stated in the banks problem;
- Investment decisions solve the problem of capital goods producers, taking price capital as given and where capital stock evolves according to: $K_{t+1} = \big[I_t + (1-\delta)\xi_t K_t\big]$
- Prices and intermediate goods demand solve competitive retailers problem;
- (Conventional) monetary policy is set according to a Taylor Rule , and the balance sheet is set such that $M_t = Q_t^B B_t^{CB}$, taxes are such that the government budget constraint is satisfied;
- All markets clear

3.6 QT Mechanisms

QT consists on the Central Bank selling government bonds while it decreases the reserves stock. The partial equilibrium effect of the bond sales via the banks is given by: $\frac{-dQ_t^B/Q_t^B}{dB_t/B_t} = -\frac{1}{\gamma^b - 1} > 0$: the stock of government bonds in hands of the financial intermediaries increase, so the price decreases.

As the leverage constraint is always binding:

$$Q_t^S S_t = \overline{\phi}_t N_t - \Delta^L Q_t^B B_t$$

Using the market clearing conditions and replacing the leverage expression we get:

$$Q_t^K K_t = \frac{\nu_t^d N_t}{\Theta\big(\frac{M_t}{N_t}\big) - \nu_t^k(1 + \Psi_t)} - \Delta^L Q_t^B \big(B^G - B_t^H - B_t^{CB}\big)$$

where $\nu_{k,t} = \mathbb{E}_t\, \Lambda_{t,t+1}\Omega_{t,t+1}(R^K_{t+1} - R^d_t)$ is the credit spread and $\nu_{d,t} = \mathbb{E}_t\, \Lambda_{t,t+1}\Omega_{t,t+1}\big(R^d_t - C_t\big)$ is the funding cost.

QT increases the amount of bonds in hands of the private sector: the leverage is higher as the swap is between a bond and reserves that are a better quality asset in terms of absconding/liquidity (have 0% absconding rate compared to Δ^L), so the constraint tightens and there is less space for credit.

Credit depends positively on net-worth and credit spreads. As the constraint gets tighter (higher $\Theta(M_t/N_t)$), capital claims decrease: this is another QT channel that operates through the drain in reserves.

Replacing the household optimal demand for government bonds:

$$Q^K_t K_t = \frac{\nu^d_t N_t}{\Theta\big(\frac{M_t}{N_t}\big) - \nu^k_t(1+\Psi_t)} - \Delta^L Q^B_t \left(B^G - \left(\overline{B}^H + \frac{\mathbb{E}_t\, \Lambda_{t+1}(R^B_{t+1} - R^d_t)}{\kappa^H} \right) - B^{CB}_t \right)$$

The level of credit is higher the higher is the maximum leverage the banks can take. The negative credit effects of QT are mitigated when the frictionless amount of bonds held by households is higher and when they face lower transaction costs (κ^H is low). When it is less costly for households to participate in the government bond market, the "credit-crunch" effect is mitigated.

The reserves convenience yield is defined as:

$$\mathbb{E}_t\, \Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\big(R^d_t - R^M_t\big) = \mathbb{E}_t\, \Lambda_{t,t+1}\Omega_{t+1}\Pi^{-1}_{t+1}\frac{\big(R^d_t - R^B_{t+1}\big)}{\Delta^L}\frac{d\Theta(M_t/N_t)}{dM}\frac{\phi_t N_t}{\Theta(M_t/N_t)}$$

The yield is stronger when the relaxation of the incentive constraint is higher and the larger is the government bond spread. Both terms are negative, so reserves have a liquidity premium over deposits.

From the credit FOC from the financial intermediaries, and using $\mu_t = \frac{\lambda_t}{1+\lambda_t}$:

$$\frac{\mathbb{E}_t(R^F_{t+1}) - R^d_t - C'_S}{R^d_t} = \frac{\Theta(\cdot)\mu_t}{\Omega_{t+1}} - cov_t(\Lambda_{t,t+1}, R^F_{t+1}) - \frac{\sigma cov_t\Big(\Theta(\cdot)\phi_{t+1}, \Lambda_{t,t+1}(R^F_{t+1} - R^d_t - C'_S)\Big)}{\Omega_{t+1}}$$

The first term represents financial frictions: this is decreasing on the tightness of the constraint via reserves injection. The second term is the risk premium term coming from the household discount factor. Finally, high excess returns reflect compensation that bankers demand for holding risk.

In the benchmark economy, the supply of government debt, B^G is fixed, but later we will perform an exercise that shows that shrinking the supply of these assets dampen the negative effects. The fiscal channel has three components affected by QT: the decrease in remittances from the central bank, the increase in automatic stabilizers/ transfers and the higher debt payments. All of them work in the same direction: a reduction in the fiscal space.

3.7 QT: Bank Mechanisms

The bank profits are defined as $\Pi_t^B = \frac{\Pi_t N_t}{N_{t-1}}$. We conduct a bank profit decomposition in a similar spirit as in De Groot and Haas (2023) to study the consequences of a reduction of the central bank's balance sheet size on the banks. The details of the derivation are in appendix 9.6.

$$\Pi_t^B = \Pi_t(R_t^F - R_{t-1}^D)\frac{Q_{t-1}^K S_{t-1}}{N_{t-1}} + \Pi_t(R_t^B - R_{t-1}^D)\frac{Q_{t-1}^B B_{t-1}}{N_{t-1}} + \Pi_t(R_{t-1}^M - R_{t-1}^D)\frac{M_{t-1}}{N_{t-1}}$$
$$+R_{t-1}^D - C(N_{t-1}, Q_{t-1}^K S_{t-1})$$

Adding and subtracting $\mathbb{E}_{t-1}\,\Pi_t R_t^F \frac{Q_{t-1}^K S_{t-1}}{N_{t-1}}$ and $E_{t-1}\Pi_t R_t^B \frac{Q_{t-1}^B B_{t-1}}{N_{t-1}}$, defining , $\Phi_t^K = \frac{Q_t^K S_t}{N_t}$, $\Phi_t^M = \frac{M_t}{N_t}$ and $\Phi_t^B = \frac{Q_t^B B_t}{N_t}$, $CS_t = \mathbb{E}_t\,\Pi_{t+1} R_{t+1}^F - R_t^D$, $TS_t = \mathbb{E}_t\,\Pi_{t+1} R_{t+1}^B - R_t^D$ and log-linearizing around the steady-state:

$$\hat{\pi}_t^B = \underbrace{\frac{R^F\Phi^K + R^B\Phi^B}{\Pi^B}\left(\hat{\pi}_t - \mathbb{E}_{t-1}\,\hat{\pi}_t\right)}_{\text{Announcement: Inflation}} + \underbrace{\frac{mpk\Phi^K}{\Pi^B}\left(\hat{mpk}_t - \mathbb{E}_{t-1}\,\hat{mpk}_t\right)}_{\text{Announcement: Dividend}}$$
$$+\underbrace{\frac{\Phi^K}{\Pi^B}\left(\hat{q}_t^k - \mathbb{E}_{t-1}\,\hat{q}_t^k\right) + \frac{\Phi^B}{\Pi^B}\left(\hat{q}_t^B - \mathbb{E}_{t-1}\,\hat{q}_t^B\right)}_{\text{Announcement: Capital Gains}} + \underbrace{\frac{CS\Phi^K}{\Pi^B}\hat{cs}_{t-1}}_{\text{Credit Spreads}} + \underbrace{\frac{TS\Phi^B}{\Pi^B}\hat{ts}_{t-1}}_{\text{Term Spread}}$$
$$+\underbrace{\frac{R^M\Phi^M}{\Pi^B}\hat{r}_{t-1}^M - \frac{R^D\Phi^M}{\Pi^B}\hat{r}_{t-1}^D}_{\text{Short Term Rates Difference}} + \underbrace{\frac{R^D}{\Pi^B}\hat{r}_{t-1}^D}_{\text{Deposit Rate}} - \underbrace{\frac{C}{\Pi^B}\hat{c}_{t-1}}_{\text{Leverage Cost}}$$

In Figure 8 we plot the profit decomposition after a one time QT: the main driver of the negative impact on profits are the capital losses on both assets, but then the profit goes above the steady state due to higher spreads. This pattern will show again the perfect foresight path dynamics.

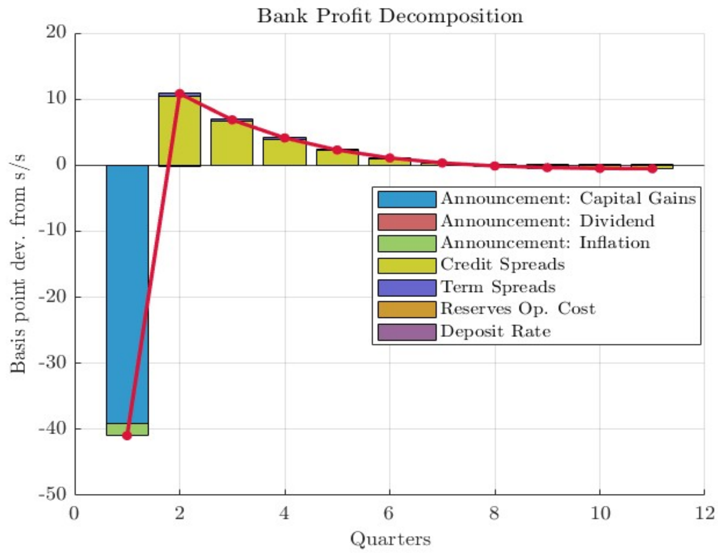

Figure 8: Banks: Profit Decomposition after One-Time QT of 1% of GDP

3.8 A Two-Period Model

In this section we study a two-period model to understand the role of reserves injection on the credit and prices of government bonds. Households supply an inelastic amount of labor, there is full depreciation of capital, no investment adjustment costs. We assume no price stickiness and no mark-ups. There are no balance sheet costs for banks. We set $\sigma = 0$. The supply of bonds will remain fixed. Finally we assume that the aggregate net-worth with which new born intermediaries start operating at the initial period is $N = X + Q^B B_{-1}$. Similar assumptions to study a simple version of this model are taken by van der Kwaak (2023) to study liquidity facilities by the ECB.

Proposition: Credit is increasing on QE: a one-time asset swap between reserves and government bonds. $\frac{dK}{dM} = \frac{dS}{dM} > 0$.

Proof: Appendix 9.16

Proposition: The price of government bonds with respect to reserves is positive:

$$\frac{dQ^B}{dM} = \underbrace{-\Delta\alpha(\alpha-1)K^{\alpha-2}Q^{B^2}}_{(+)} \underbrace{\frac{dK}{dM}}_{(+)} > 0$$

Proof: Appendix 9.16

Proposition: The convenience yield is positive and decreasing on reserves:

$$(R^D - R^M) = \underbrace{(R^D - R^B)}_{(-)} \underbrace{(S + \Delta^L Q^B B)}_{(+)} \underbrace{\frac{\Theta'(M/N)}{\Delta^L \Theta(M/N)}}_{(-)} > 0$$

Proof: Appendix 9.16

Proposition: Initial period bank net worth with respect to reserves is:

$$\frac{dN}{dM} = \frac{dQ^B}{dM} B_{-1} > 0$$

Proof: Appendix 9.16

4 Calibration

In this section, we proceed with the model parameterization. The discount factor β is set at 0.995 to match a deposit interest rate of 2%. We normalize the long-term government bond coupon to 1, while the Woodford decaying factor is set at 0.975 to match a duration of 8 years. The calibration of the production side parameters follows standard practices. For the bank parameters, we set $\theta = 0.54$, $\Delta^L = 0.33$, and the survival rate $\sigma = 0.95$ to jointly match a leverage of 4, a credit spread of 275 basis points, and a term spread of 100 basis points. The steepness of the reserves demand, γ, is set at 0.025 to match a difference of 12 basis points between the 3-Month or 90-Day Rates and the yields on Certificates of Deposit and interest on reserves. Finally, κ^L is set based on Ulate (2022), which conducts regressions for US banks to estimate the parameter, specifically how a 1 percent change in the capitalization ratio affects loan rates.

Table 5: Model Calibration

Parameter (1)	Value (2)	Description (3)	Target/Source (4)
		A. Utility	
β	0.995	Household discount factor	$R^D = 2\%$
ϕ	1	Relative risk aversion	Standard log utility
ψ	2	Inverse Frisch elasticity	Sims and Wu (2021)
χ	3	Disutility of Labor	SS Labor 1
c	1	Coupon Long-Term Bond	Standard
κ^H	1	HH Debt Elasticity	GK2013
γ^b	0.975	Woodford decaying coupon	Duration of Long-Term Treasury 10 ys
		B. Production/ NK Block	
ϵ	11	Production Elasticity	Sims and Wu 2021
δ	0.025	Capital depreciation rate	GK2013
ϕ^P	118	Rotemberg Adjustment Cost Parameter	Consistent with Calvo Frequency 0.75
α	0.33	Share Capital Production Function	Standard
ϕ^k	2	Adjustment Cost parameter Capital Producers	GK2013
		C. Policy	
ϕ_π	1.25	Response to inflation MD	Standard Monetary Dominance
ϕ_y	0.33/4	Response to output gap	Standard
θ^G	0.27	Fiscal Stimulus	Bianchi-Melosi 2017
ρ_i	0.7	Policy smoothing	Standard
G	0.2	SS government spending	G/Y
b^G	0.6	SS government debt	$\frac{B^G Q^B}{4Y}$
		D. Banks	
θ	0.54	Parameter I Bank Constraint	$R^M - R^D$ -12 Basis Points
γ	0.025	Parameter II Bank Constraint	$R^F - R^M$ 275 Basis Points
Δ^L	0.33	Liquidity Government Bonds	$R^B - R^M$ 100 Basis Points
σ	0.95	Bankers survival rate	GK2013
κ^L	0.005	Leverage Costs	Ulate (2022)
X	0.12	Transfer to bankers	

Now, we proceed to compare ratios of deposits over liabilities and the corresponding ratios for reserves. Also we compare the spreads between the model and data. The second moments (volatility and output correlation) are also computed. [8]

[8] Here we use the FFR for the short term in the data. For the model moments, the interest on reserves is used to compute the term spread.

Table 6: Bank Asset-Liabilities compositions and interest rate spreads: Steady-State

Variable (1)	Label (2)	Model (3)	Data (4)
	A. Banks: Non-Targeted		
D/TBL	Deposits over Liabilities	75%	77%
M/TBL	Reserves over Liabilities	4.5%	6.6%
M/TBA	Reserves over Assets	5.2%	5.7%
	B. Interest Rate Spreads: Targeted		
R^F-R^M	Loans Spread	275 bp	275 bp
R^B-R^M	Government Bonds Spread	100 bp	100 bp
R^M-R^D	Convenience Yield	-12 bp	-12 bp

Table 7: Model vs Data: 1980Q1: 2019Q4

Variable	$\Delta \ln Y_t$	$\Delta \ln C_t$	$\Delta \ln I_t$	$\Delta \ln L_t$	$\Delta\pi_t$	CS
			Volatility			
Model	0.68	0.45	2.24	1.36	0.21	0.024
Data	0.68	0.49	2.64	3.68	0.43	0.23
			Cyclicality			
Model	–	0.44	0.53	0.22	0.006	-0.27
Data	–	0.59	0.87	0.14	-0.09	-0.46

The regression $EFFR_t - IOR_t = \alpha + \beta \log(M_t) + \epsilon_t$ is conducted for both the data outside the ZLB between 2013 and 2020 and the model, which computes the regression using the outcomes from 10,000 stochastic simulations. The results are presented in Table 8:

Table 8: Regression Liquidity Effects: Data (2013-2020) vs. Model

	Data	**Model**
Coefficient	-0.226^{***}	-0.224^{***}
	(0.017)	(0.012)

The regression conducted aligns with the weekly average from the NY Fed's recent 'Reserve Demand Elasticity' series, which shows that the elasticity of the federal funds rate to reserve changes is very small and statistically indistinguishable from zero from 2010 to 2024. This suggests that our steepness parameter in $\Theta()$ remains valid across both QT episodes.

As the demand curve for reserves makes the model highly non-linear, the starting point of the balance sheet size when unconventional policy is conducted matters, and there is an asymmetry between QE and QT. They key parameter will be the one governing the reserve demand, γ. We study the multipliers using a second order approximation. The effects on the financial variables as net growth, bank funding costs and credit are higher in QT when the reserve demand curve is steeper. The cumulative is for the first four quarters, computed as the present-value multiplier (a similar concept that the fiscal multiplier in Mountford and Uhlig (2009)) for the one-time QT:

$$PVM = \frac{\sum_{i=1}^{T} \beta^{i-1} \Delta Y_{t+i}}{\sum_{i=1}^{T} \beta^{i-1} B_{t+i}^{CB}}$$

,

Table 9: QE/QT Multipliers: 1% Balance Sheet Reduction

	Y	Inflation	NW Growth	Credit Growth	Rd-Rm
			QE		
Impact $\gamma = 0.015$	0.31 %	0.1%	4.27%	1.43%	−5.6 b.p.
Cumulative $\gamma = 0.015$	0.46 %	0.06%	−0.20%	0.57%	–
Impact $\gamma = 0.025$	0.34 %	0.11%	4.49%	1.5%	−9.5 b.p.
Cumulative $\gamma = 0.025$	0.5 %	0.07%	−0.08%	0.61%	–
			QT		
Impact $\gamma = 0.015$	−0.24 %	−0.15%	−4.67%	−1.69%	9.8 b.p.
Cumulative $\gamma = 0.015$	−0.55 %	−0.12%	1.87%	−0.7%	–
Impact $\gamma = 0.025$	−0.27 %	−0.17%	−5.28%	−1.88%	18 b.p.
Cumulative $\gamma = 0.025$	−0.61 %	−0.13%	1.99%	−0.76%	–

The model is able to generate asymmetries between QT and QE. Recently Cantore and Meichtry (2023) found state-dependency, defined as being at or close to the ZLB as a source of asymmetry in the multipliers. Our model also captures this state-dependency: the QE effect on output at the ZLB is approximately 3.5 higher than outside the ZLB.

5 Quantitative Analysis

5.1 Tightening in the model: Interest Rate vs One Time Reduction of the Balance Sheet Size

Before studying a perfect-foresight non-linear path to compare different strategies of QT, we start by inspecting the mechanism of a one-time QT against a short term interest rate increase on two sets of variables: the financial variables, as studied in the empirical evidence section, and then on the main macro variables. We conduct a second-order perturbation. Calibrating the size of the QT shock to match the same output growth impact, we plot the IRFs of both QT and rate hiking. Interest rate is more effective to manage inflation, but net-worth is more reactive to balance sheet movements. This distinction will be crucial for the optimality exercises.

Wei (2022) studies in a Vayanos-Vila framework situations where both policies deliver the same outcome on the 10 year yield bond.

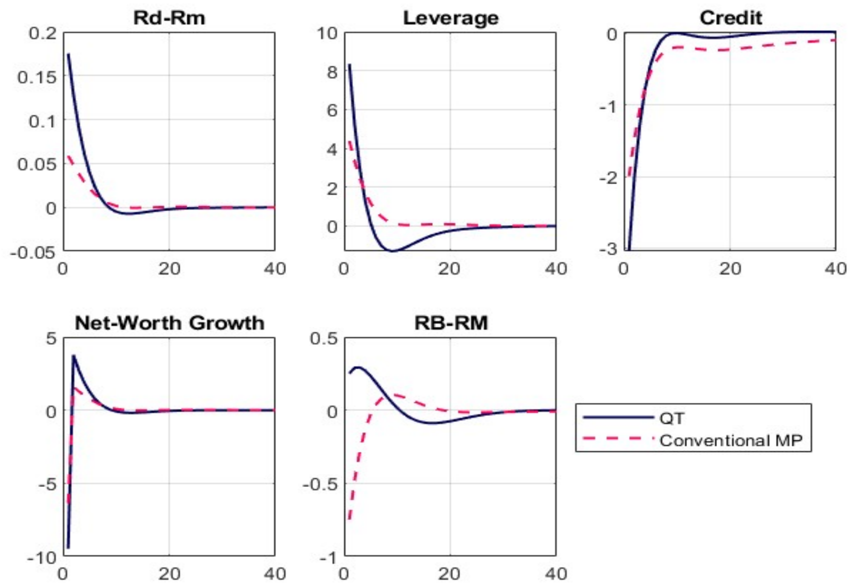

Figure 9: Impact on Financial Variables (in % deviation from SS)

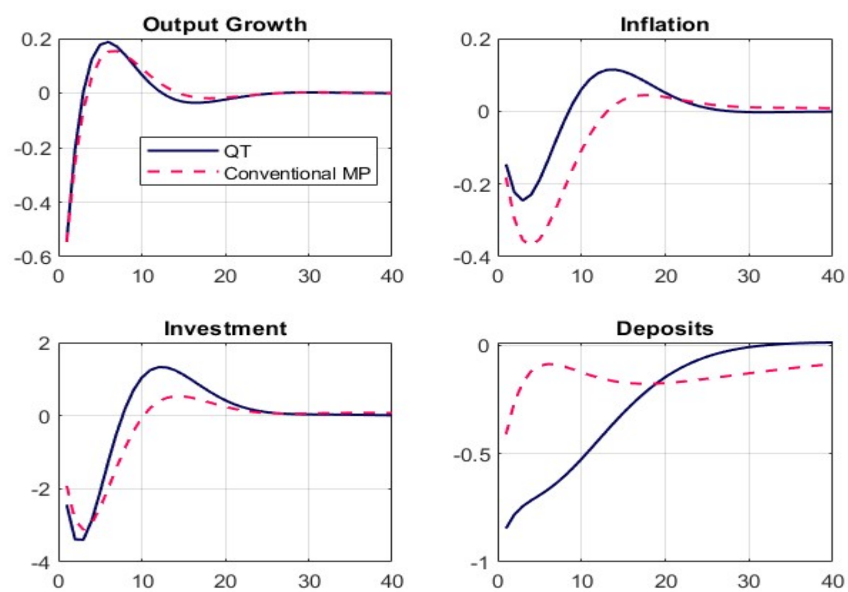

Figure 10: Impact on Macro Variables (in % deviation from SS)

5.2 Transitional Dynamics along a Perfect-Foresight Path

Now we will continue with a full non-linear transition dynamics when the central bank conducts QT reaching a lower steady-state size of the balance sheet. We solve the model in a non-linear way to take into account non-linearities and non-monotonicities. The rational-expectations nonlinear system of equilibrium conditions at time t can be expressed as: $\mathbb{E}_t\left[f(y_{t+1}, y_t, y_{t-1}, u_t)\right] = 0$. The information set of the agents at time t includes the sequence of shocks $\{u_t\}_t^T$, only shocks at t are unexpected. Details of the non-linear solution method as a two-boundary value problem along a perfect foresight path are in appendix 9.14. The benchmark unwinding scenario features a 30% reduction of the balance sheet size. We plot the credit growth, the liquidity costs, leverage, the term spread, output growth, the policy rate, the central bank profits, the growth of banks net worth, inflation, deposits, and the term premium. The latter is defined as the ratio between the yield to maturity of the long term bond and the hypothetical expectations hypothesis bond, where yields to maturity are $R_t^{B,Y} = \frac{1}{Q_t^B} + \gamma^b$. [9] The reduction in the balance sheet size translates into a decrease in credit due to the lower net-worth and lower bank's credit space, increase in the short term funding costs and a decrease in output due to lower investment. The increase in the long-term yields makes the term spread and premium increase. Deposits decrease due to a standard substitution effect from households.

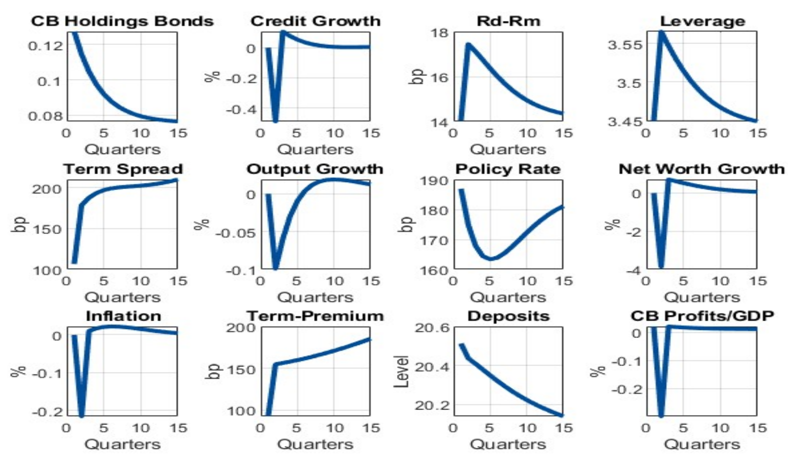

Figure 11: Dynamics after a BS reduction

Now we proceed to compare the reduction of the balance sheet process under some key parameter changes. The most important parameter is the one measuring the liquidity benefits of reserves: when this parameter increases, the constraint gets tighter at a higher pace

[9]The EH bond price expression is $Q_t^{EH} = \frac{1+\gamma^b\,\mathbb{E}_t\,Q_{t+1}^{EH}}{R_t^d}$

the lower are the reserves stock, so QT is more contractionary. We increase the parameter to $\gamma = 0.04$ to match a higher SS difference between reserves and deposit rates of 25 bp . The demand for government debt is the other key variable: when the households can buy this assets at lower costs (lower κ^H, or elastic) they will absorb more debt, so the bank lending channel is weaker.

Another coordinating policy that might dampen the negative effects of QT is decreasing the supply of government debt. Wright (2022) claims that it's an equivalent policy. Another mitigating policy was the Liquidity Facilities designed by the Federal Reserve. We study this policy in detail in appendix 9.4.

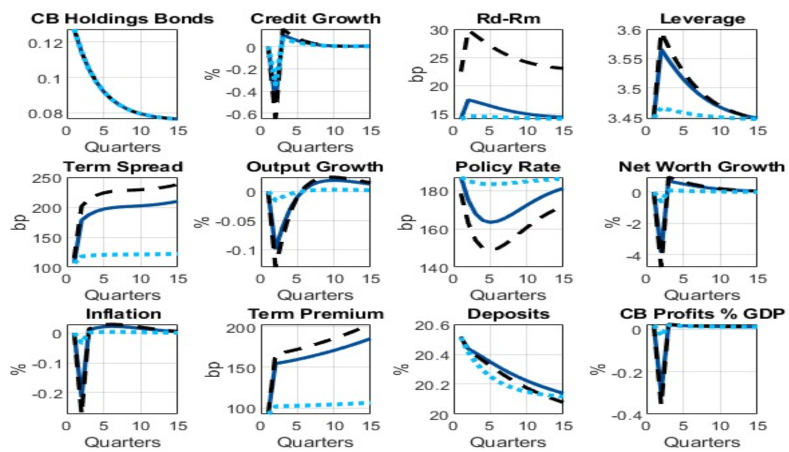

Figure 12: Dynamics after a BS reduction. The blue line is the benchmark scenario. The black dashed line plots an increase in liquidity benefits of reserves such that $\gamma = 0.04$. The light blue dotted line is a scenario where the debt supply shrinks at the same rate of QT.

We proceed to plot the evolution of credit both under the baseline economy and an alternative scenario in which households can access the long-term bonds market under very low costs. In the latter case, the credit decrease of a QT program is mitigated due to a lower rebalancing channel.

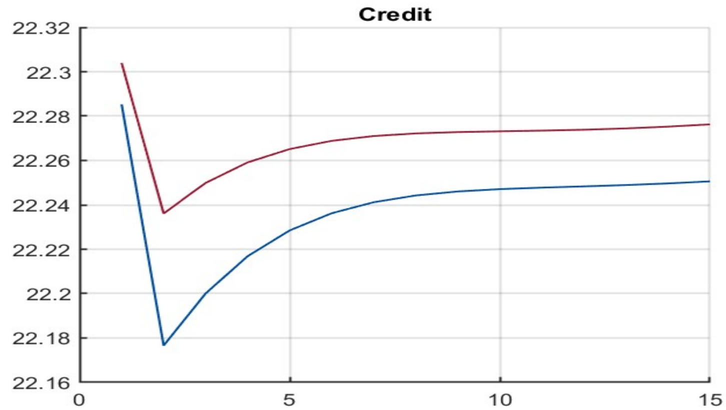

Figure 13: Evolution of credit. The blue line is the benchmark economy. The red line shows the evolution of credit when household can access long-term bonds under low costs ($\kappa^H = 0.05$)

5.3 QT Announcement Effects

Until now, we assumed perfect-foresight paths in which agents learnt all the information regarding the unwinding in the initial period. Now, we will explore how the outcomes change when the announcement is close to the beginning of QT. The Central Bank starts to conduct the unwinding process at t=5. The blue line reflects the outcome when the policy is announced 4 periods before, while the black line represents the policy outcomes when the announcement is made at the same time the policy is conducted. The difference in the outcomes is due to announcement effects. Announcing QT with enough anticipation yields better macro-financial outcomes. In appendix 9.7 we show that when banks balance sheet costs and when reserves do not have liquidity benefits, the differences in outcomes for different announcement timings are much smaller

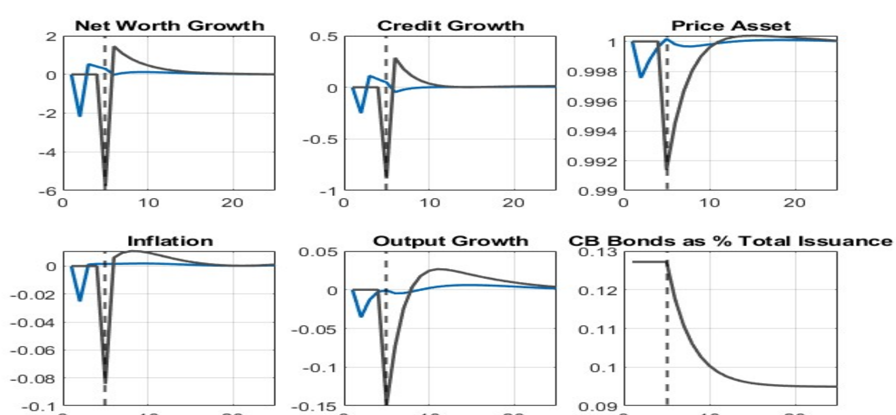

Figure 14: Announcements: The blue line represents the announcement made 4 quarters before the start of the policy, while the black line reflects the outcome of the announcement with only 1 quarter of anticipation.

5.4 A Crisis simulation: QE response and the subsequent QT strategies

Here we will simulate a crisis where an unexpected adverse capital quality shock hits the economy, from periods 1 to 3. The central bank increases the size of the balance sheet reaching a peak at t=6, and then proceeds with different strategies: passive unwinding or active sales.

In Figure 15 the effect of QE is studied: the policy rate lift off occurs one year before, while the capital price, inflation and output growth recover faster. Figure 16 shows a trade-off between sales and a passive unwinding strategy: sales mitigate the output and price drop at the beginning since agents expect a ”shorter-lived” stimulus, so they have incentives to work more hours and firms to conduct investment decisions as interest rate will be higher in the close future. But when sales are actually conducted, implementation effects arise and the output growth is below when sales are actually conducted. Figure 17 shows the dynamics of a more extreme scenario of a one-time sell-off, where output growth remains considerably below the passive unwinding from periods 4 to 9. In the case of unexpected one-time sell-off, a double-dip recession arises in line with findings of Foerster (2015). The figure of this scenario is in appendix 9.8. Finally, Figure 18 shows the difference between announcing unwinding and then conducting sales against an strategy where sales are announced but then a slower pace policy is conducted. In terms of volatility, and impact mitigation, the latter policy is better.

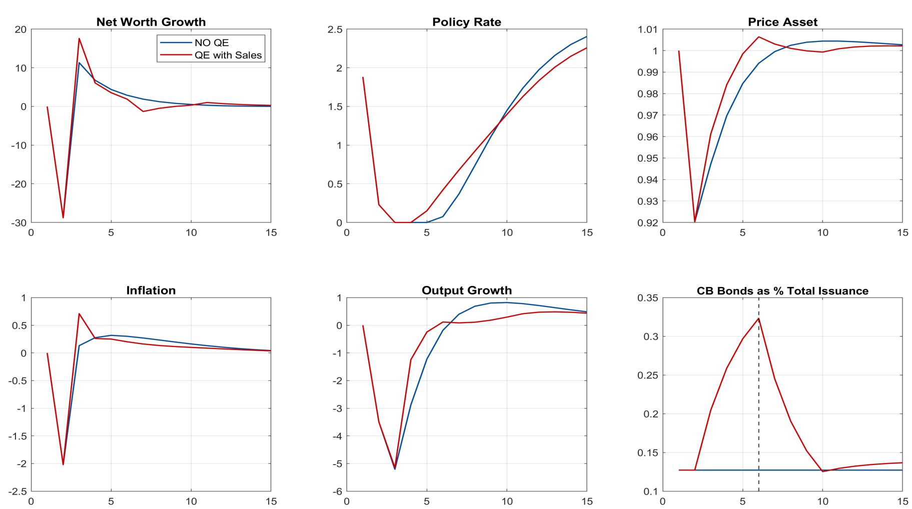

Figure 15: Crisis Event: QE vs No Intervention. The blue line is the scenario without policy intervention and the red one plots the variables with QE intervention.

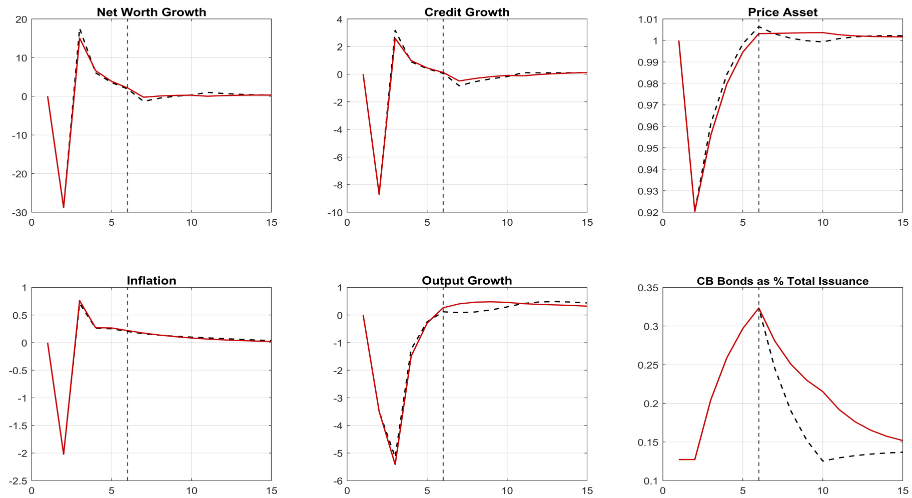

Figure 16: Crisis Event: Sales vs Passive Unwinding. The black lines plot the sales scenario while the red ones the passive unwinding one.

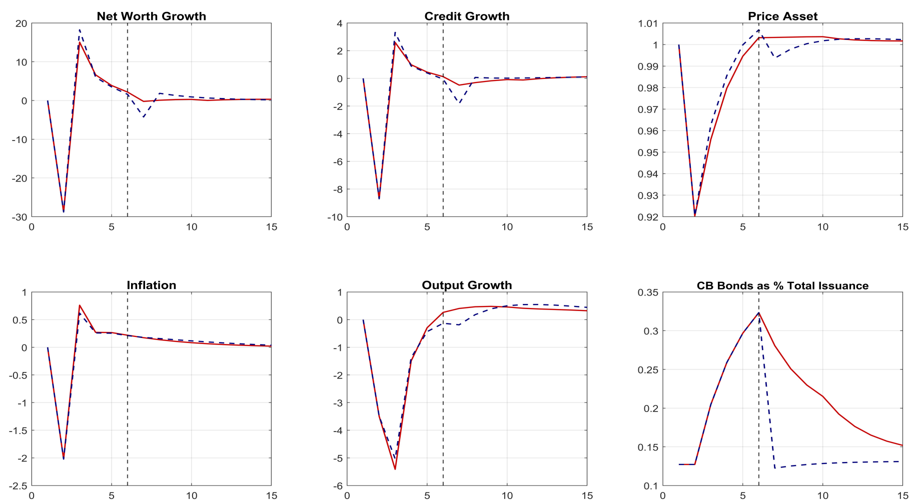

Figure 17: Crisis Event: One-Time Sell-Off vs Passive Unwinding. The blue dashed lines plot the sales scenario while the red ones the passive unwinding one.

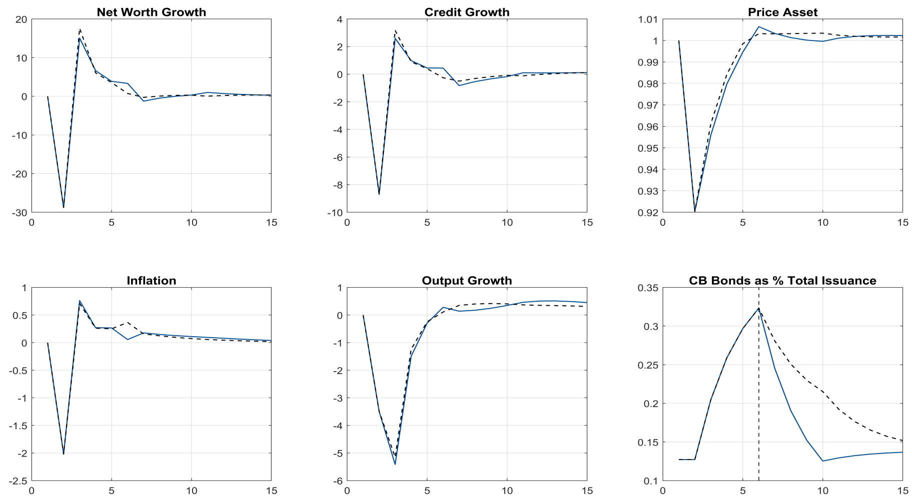

Figure 18: Crisis Event: Combination of Announcement and Implementation Strategies. Black dashed lines are the announcement of Sales and then the implementation of P.U. The blue line is the opposite scenario.

We will compare the policies in terms of output volatility and welfare. The discounted volatility of the output can be expressed as:

$$\mathcal{V}_{t,j} = \sqrt{\sum_{t}^{t+h} \beta^t \mathbb{E}_t \left(x_{t,j} - \bar{x}\right)^2} \tag{20}$$

The welfare is computed as:

$$\mathcal{W}_j = \sum_{t=0}^{\infty} \beta^j \left[\frac{C_{j,t}^{1-\phi}}{1-\phi} - \frac{\chi}{1+\psi} L_{j,t}^{1+\psi} \right] \tag{21}$$

Let λ^{CE} denote the consumption equivalent that the representative agent would need to be indifferent to stay in an economy without policy intervention, called "NOQE".

$$\begin{aligned} \mathbb{E}(V_t^{NOQE}(\lambda^{CE})) &= \mathbb{E}\left[\ln(C_t(1+\lambda^{CE})) - \frac{\chi}{1+\psi} L_{j,t}^{1+\psi} + \beta\, \mathbb{E}_t\, V_{t+1}^{NOQE}(\lambda^{CE}) \right] \\ &= \frac{\ln((1+\lambda^{CE}))}{1-\beta} + \underbrace{\mathbb{E}\left[\ln(C_t) - \frac{\chi}{1+\psi} L_{j,t}^{1+\psi} + \beta\, \mathbb{E}_t\, V_{t+1}^{NOQE}(\lambda^{CE}) \right]}_{\mathbb{E}(V_t^{NOQE})} \end{aligned}$$

Now we solve for λ^{CE} that equates the expected welfare in both economies. For the logarithmic case:

$$\lambda^{CE} = \exp\left[(1-\beta)(\mathbb{E}(V_t^j) - \mathbb{E}(V_t^{NOQE}))\right] - 1$$

The consumption equivalent measures for unwinding and sales are 0.024% and 0.021% respectively. The one for the one-time sell-off is 0.019%.

Now we compare policies when the information set changes at the time of the QT starts as the central bank surprises agents with another strategy: the measures for announcing unwinding and changing to sales, and the reverse policy are 0.019% and 0.026% respectively.

When it comes to output volatility, announcing and implementing a passive unwinding policy, or announcing sales followed by passive unwinding, yields the most stable outcomes. In contrast, the policy that generates the highest volatility involves announcing passive unwinding but then abruptly shifting to sales. One-time sell-offs are also among the worst-performing policies in terms of volatility, with unexpected sell-offs causing the greatest variance. Surprising agents with sudden sales produces the most adverse effects on output stability.

Table 10: Macroeconomic volatility

QT Strategy	Output Volatility
NO QE	1
Unexpected One-Time Sell-Off	0.922
Ann. P. Unwinding then Sales	0.912
One-Time Sell-Off	0.908
Sales	0.89
Passive Unwinding	0.881
Ann. Sales then P. Unwinding	0.86

5.4.1 Permanent QE and the importance of an Exit Strategy

Since asset sales offer a short-term stimulus but lead to poorer performance compared to passive unwinding, this section focuses on a permanent reinvestment strategy. Without the announcement or implementation of QT, net worth experiences the lowest growth due to compressed term and credit spreads during the transition. This results in higher leverage and reduced credit space from banks. The next figure illustrates the net worth difference between a passive unwinding strategy and permanent reinvestment (no QT). In terms of welfare, the difference is 0.023%. Sales also yield positive welfare gains compared to permanent reinvestment, with an improvement of 0.02%.

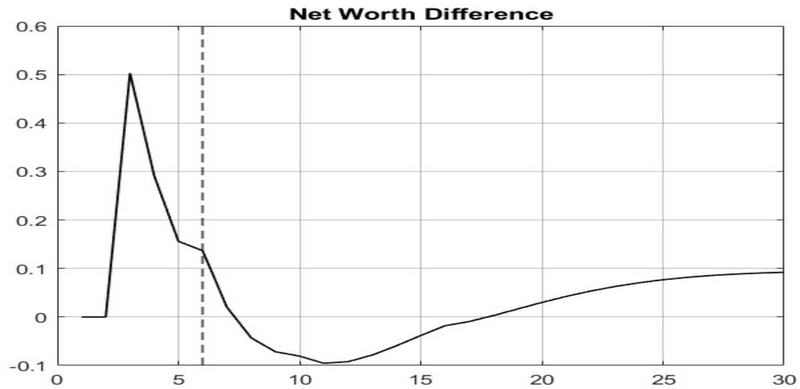

Figure 19: Net Worth evolution. Difference between a passive unwinding strategy and a permanent reinvestment strategy

5.5 Introducing a cap to Central Bank Purchases: The role of Policy Room

Now we proceed to introduce an (occasionally) binding constraint to the size of the central bank: it is constrained in the purchases that it can conduct. We set it at 35% of the total issuance.

$$B_t^{CB} = \min\left\{(1-\rho_{CB})B^{CB} + \rho_{CB}B_{t-1}^{CB} + \sum_{j\geq 0}^{T}\epsilon_{t|t-j}^{CB}, \overline{B}^{CBmax}\right\}$$

We conduct the following exercise:

1. The CB conducts Unwinding or Sales. It announces in the first period that in t=3 it will start the policy.
2. At t=11 an unexpected financial crisis hits the economy.
3. The CB conducts QE starting in t=11.

As we can see from the figure, with an unexpected financial crisis during an unwinding process, the policy room, the difference between the cap and the current size of the central bank balance sheet, matters as a more aggressive balance sheet expansion attenuates the credit and net worth growth and makes output recovers faster.

Here we compute welfare in terms of the steady state:

$$\lambda^{CE} = \exp\left[(1-\beta)(\mathbb{E}(V_t^j) - \mathbb{E}(V_t^{SS}))\right] - 1$$

The ex-ante welfare of the passive unwinding policy is higher than the sales policy: for passive unwinding -0.0107% against -0.0112% for sales. But the ex-post welfare, after the announcement of the policy occurring one period after the crisis, is higher for sales due to the more aggressive QE/higher policy space: -0.01903% and -0.0188% respectively.

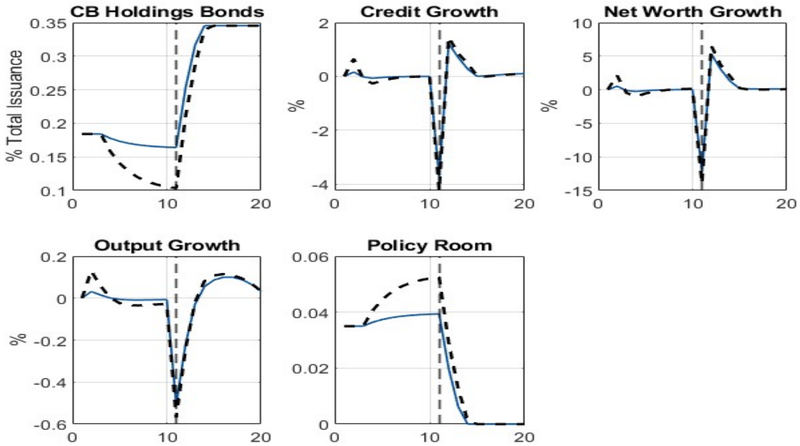

Figure 20: The Policy Space. The blue line is the outcomes of the passive unwinding scenario while the black one is sales

6 Constrained Optimal Policy Projection

In this section we will perform optimality analysis under a Constrained Optimal Policy Projection approach, following de Groot et al. (2021). This tool solves for optimal paths, taking a baseline scenario and adding optimal sequences of shocks that minimize a loss function. Optimal policy projections are based on three elements: a baseline scenario that is the market or policymaker scenario forecast of macro variables at the time of the shock. We will take the Great Financial crisis and the Greenbook/Tealbook projections at the time of the beginning of the crisis. Second, we need IRFs from a structural model. We will proceed with the model of the previous section. Finally, we describe the loss function of the policy-maker, that minimizes a quadratic loss function that depends on inflation deviations , output gap, and the policy interest rate deviations.

In additional exercises we extend the loss function with two additional balance sheet terms: one of the size of the balance sheet and the second of deviations reflecting that sudden changes in bond sales can have financial stability implications. Harrison (2024) shows that these two terms are meaningful when there are portfolio frictions or intervention costs. In this case, the two additional terms are $\lambda_{BS}(Q_t^B B_t^{CB})^2 + \lambda_{\Delta_{BS}}(Q_t^B B_t^{CB} - Q_{t-1}^B B_{t-1}^{CB})^2$.

$$\mathcal{L}_t = \min_{R_t^M, B_t^{CB}} \frac{1}{2}\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ (\Pi_t - \Pi)^2 + \lambda_y (Y_t - Y)^2 + \lambda_r (R_t^M - R^M)^2 \Big\} \quad (22)$$

The weights are standard ones used in the optimal policy literature: $\lambda_y = 0.125$ and $\lambda_r = 4$.

The planner will face two constraints in the two instruments: the ZLB for the interest rate, and a cap for the balance sheet size of 0.7 of the total government bond issuance.

The problem can be characterized as a linear quadratic problem in matrix terms:

$$\min_{\epsilon_t} \frac{1}{2}\left\{ Z_t' \underbrace{W}_{\text{Pref. Matrix}} Z_t \right\}$$

$$\text{s.t.} \quad \underbrace{Z_t}_{\text{Optimal Policy Path}} = \underbrace{B_t}_{\text{Baseline}} + \underbrace{D\epsilon_t}_{IRFs}$$

$$B_t^{R^M} + D^{R^M}\epsilon \geq 1 \quad \text{(ZLB)}$$

$$-D^{B^{CB}}\epsilon_t \leq B_t^{B^{CB}} \quad \text{(Balance Sheet Lower Bound)}$$

$$B_t^{B^{CB}} + D^{B^{CB}}\epsilon \leq \bar{B}^{CB} = 0.7B^G \quad \text{(Balance Sheet Upper Bound)}$$

$$\epsilon_t \equiv \begin{pmatrix} \epsilon_t^{R^M} \\ \epsilon_t^{CB} \end{pmatrix}$$

To dampen the forward guidance puzzle, as in de Groot and Mazelis (2020), we will use that some agents, 30% of them, are inattentive. In the Appendix 9.18 we provide details of the method, and the algorithm. We discuss in Appendix 9.19 details on finite planning, expectations/agent attention and credibility. Also, we provide robustness check the numerical values for the weights in the loss function. [10]

The optimal QE shows that the balance sheet expansion should have been more aggressive. Under discretion, the balance sheet peak is higher and the unwinding is more gradual: this is due to the lack of a commitment device that allows to exploit the forward guidance benefits.

[10] We also analyze a case where the policymaker conducts Average Inflation Targeting, following FRBNY DSGE model, where the Taylor Rule is now:

$$R_t^M = \max\left\{\rho_R R_{t-1}^M + (1-\rho_R)\left[\phi_\pi^{AIT}(1-\rho_\pi)\tilde{\pi}_t + \phi_y^{AIT}(Y_t - Y^*) + \epsilon_t^M\right], 1\right\}$$

where $\tilde{\pi}_t$ is the discounted sum of past deviations between inflation and the target: $\tilde{\pi}_t = (\pi_t - \pi^*) + \rho_\pi \tilde{\pi}_{t-1}$ De Fiore provide details on how to implement AIT in NK models

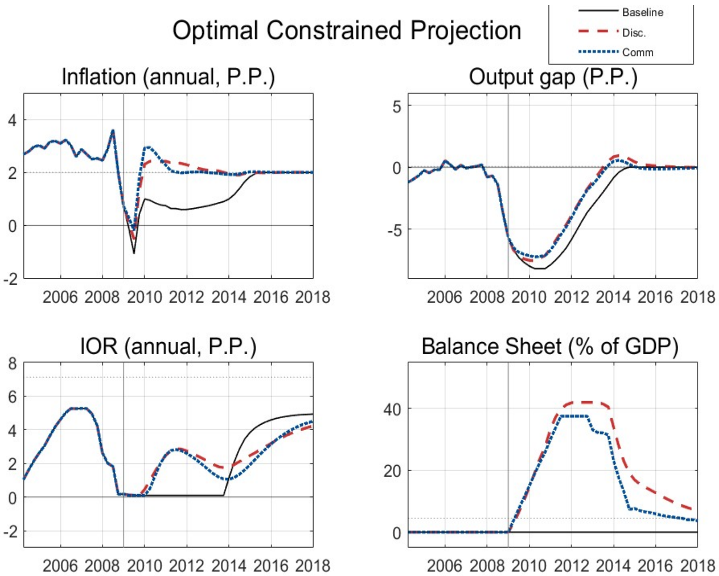

Figure 21: Constrained Optimal Policy Projection. The black line is the baseline scenario. The blue dotted line is the commitment case and the red dashed line is the discretionary scenario. Inflation, Output-Gap and the Policy Rate are expressed in P.P.

The welfare losses compared to the baseline that is normalized to 1 are 0.32 for discretion and 0.23 for commitment. When only the interest rate is used, the relative losses are 0.88 and 0.79 respectively.

The welfare losses in a limited commitment scenario are shown in the following table. When the planner has both instruments and given a full commitment benchmark of 56 quarters, the same welfare losses are attained when the planner can commit for half this horizon. (28 Q). For shorter horizon commitments, the welfare losses increase exponentially

Limited Commitment Quarters	**W.Losses Relative to Full Commitment (T=56)**
1	1.39
4	1.28
8	1.05
14	1.02
28	1.00

Table 11: Welfare Losses: Limited Commitment

Now we proceed to show the scenario where only the interest rate is used by the planner as a policy tool:

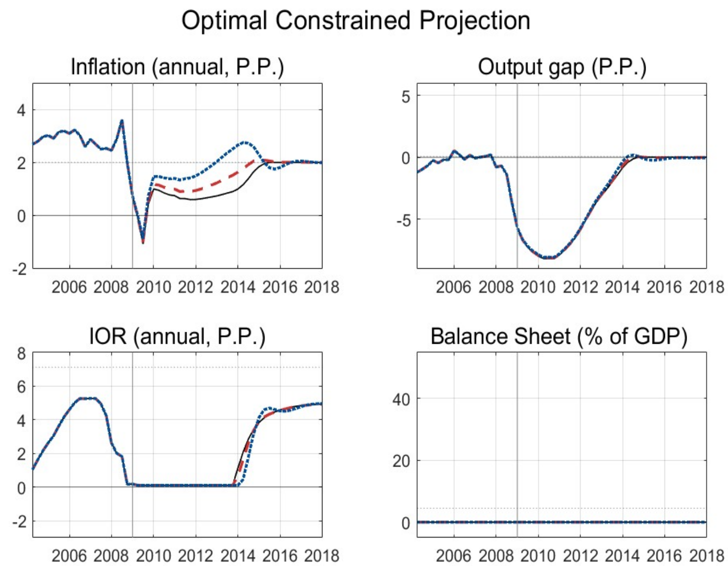

Figure 22: Constrained Optimal Policy Projection. The black line is the baseline scenario. The blue dotted line is the commitment case and the red dashed line is the discretionary scenario. Inflation, Output-Gap and the Policy Rate are expressed in P.P.

Next, we compare the QT strategies for different scenarios. First, we pick the period where QT starts for each strategy. We compare the two original scenarios of discretion and commitment, with two additional discretionary policies: one with lower debt maturity and one with a steeper reserves demand curve. The QT pace is slower with discretion and with higher reserves liquidity benefits. The lower is the debt maturity, as the monetary transmission is mitigated, the lower the optimal QT pace.

The maturity of the government bonds, captured by γ^b is $\frac{1}{1-\beta\gamma^b}$, and it creates an amplification effect compared to one period bond. From the bank's bonds optimality condition we have that:

$$Q_t^B = \tilde{\Omega}_{t+1}(1+\gamma^b Q_{t+1}^B) = \sum_{j=1}^{\infty} \tilde{\Omega}_{t,t+j}(\gamma^b)^{j-1} \tag{23}$$

where $\tilde{\Omega}_{t+1} = \frac{\Pi_{t+1}^{-1}\Lambda_{t,t+1}\Omega_{t+1}}{\frac{\lambda_t}{1+\lambda_t}\Delta\Theta(M_t/N_t)+\Pi_{t+1}^{-1}\Lambda_{t,t+1}\Omega_{t+1}R_t^d}$

The higher maturity generates larger responses to interest rate and QE/QT changes as contractionary shocks create capital losses from bonds that reduce banks net-worth and increases the tightness of the constraint. This make $\tilde{\Omega}_{t+1}$ lower and a higher maturity of debt amplifies this effect. A similar analysis for bank-runs amplification with long term bonds in a Gertler-Karadi-Kiyotaki setting is conducted by Miao, Shen, and Su (2023).

Figures 23 and 24 show the Unwinding processes for the discretion, commitment, discretion and lower maturity and reserve demand sensitivity. The first one shows the balance sheet in terms of GDP and the second is the stock, $Q_t^B B_t^{CB}$. Finally, we compare the quarterly growth rates in Figure 25.

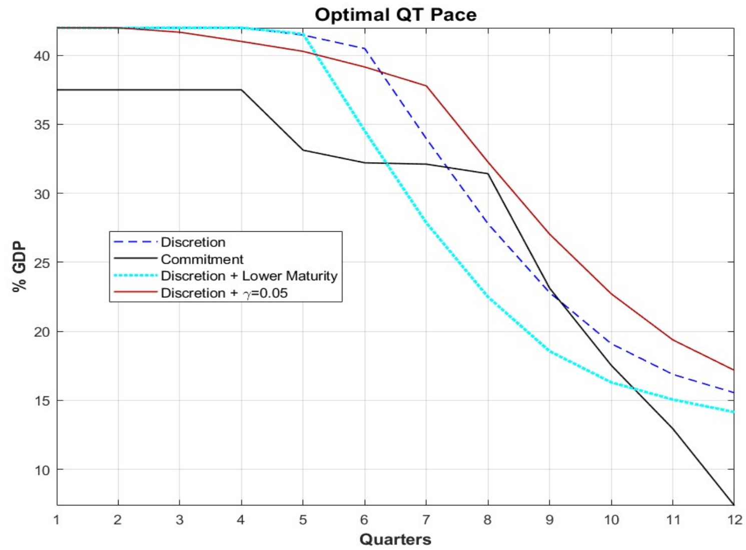

Figure 23: QT Optimal Paces: Balance Sheet as Percent of GDP

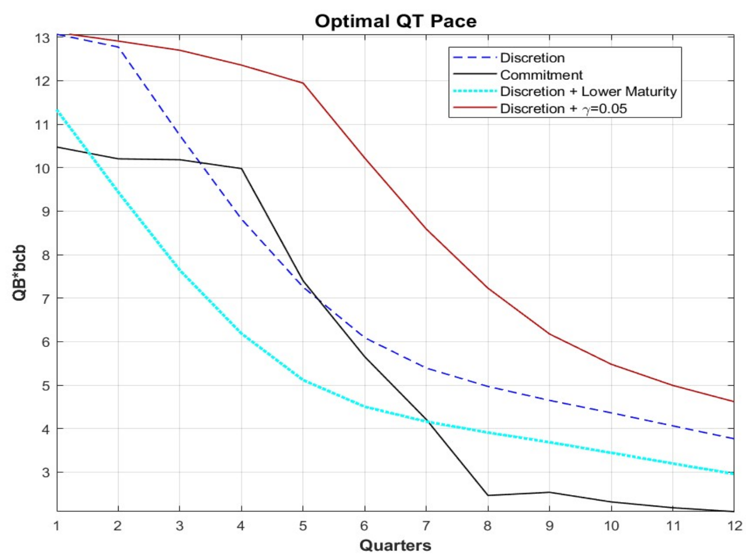

Figure 24: QT Optimal Paces: Size of the Balance Sheet

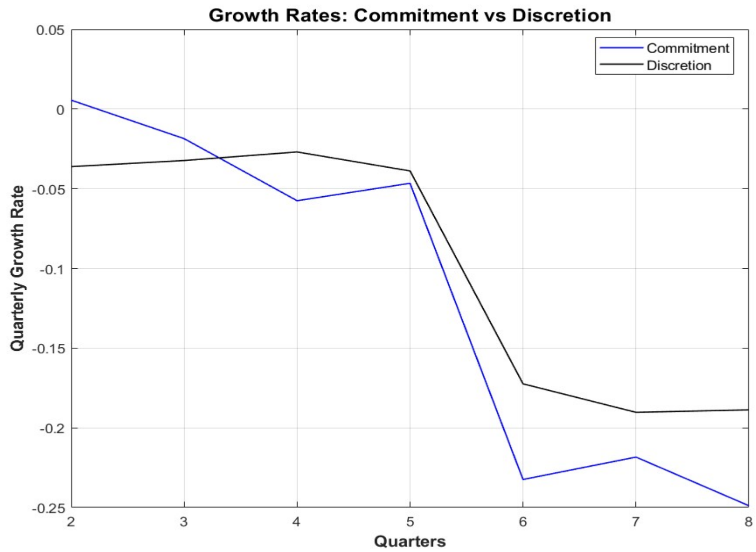

Figure 25: QT Optimal Paces: Quarterly Growth Rates Balance Sheet

The above analysis takes as the baseline the monetary authority projection, and computes deviations according to it. In Figure 31 we compute both commitment and discretionary optimal policies from the model projection and compare with the data. Optimal policies create short-lived inflation at the beginning and then stabilize at the target, avoiding all deviations. They also close the output gap before until stabilization.

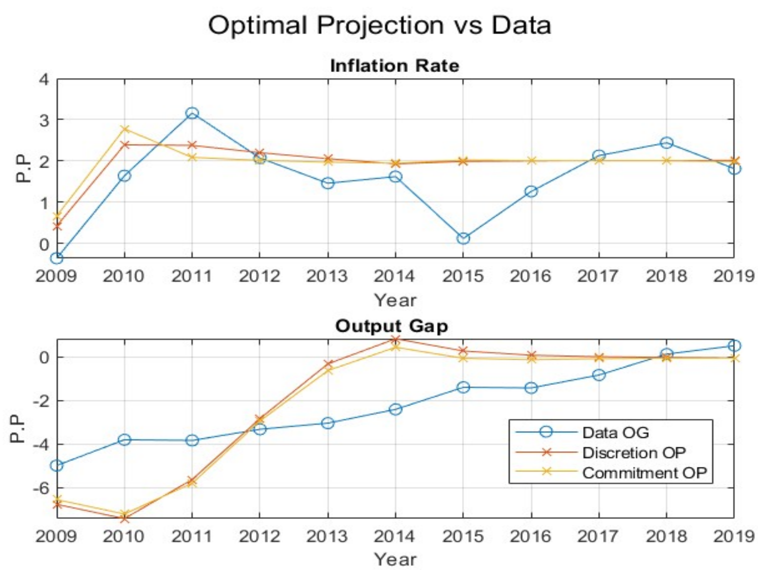

Figure 26: Data and Constrained Optimal Policy Projection

Next, we show the credit evolution under the baseline scenario and compare it with both the discretion and commitment cases:

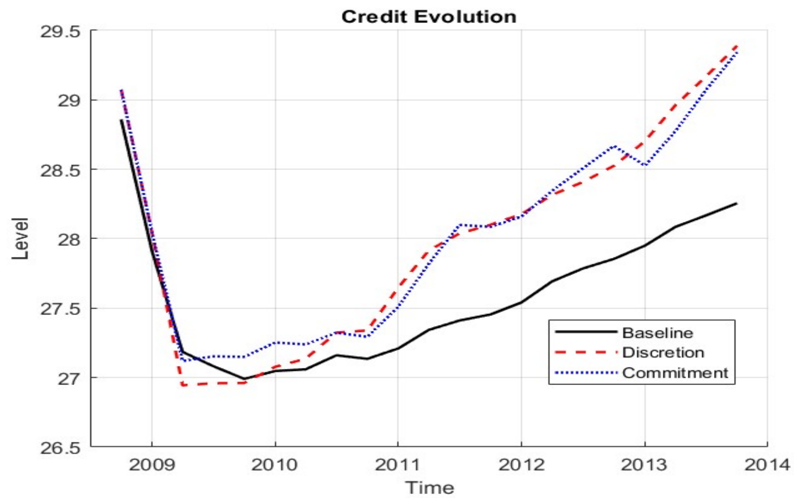

Figure 27: Credit Evolution

In the table 12, we compare the mean QT pace for the first three years after reaching the maximum size of the balance sheet for different settings, taking the discretion as a benchmark. We compare the commitment scenario, a scenario with lower average maturity and when reserves have higher liquidity benefits.

QT Strategy	**Mean(Strategy Pace)/Mean(Discretion Pace)**
Commitment	1.18
Lower Maturity: 1 Year	1.07
Higher Reserves Liq. Benefits $\gamma = 0.05$	0.9

Table 12: Relative Mean QT Pace over three years

In the commitment scenario with full attention, QT can be conducted 16% faster for the first three years than in the baseline scenario.

Regarding finite planning against the full planning: for one year, the relative losses are 0.7 and for four years 0.96.

The figure 29 illustrate the role of average debt duration and the steepness of the reserve demand curve. When both are higher, a one-time QT is amplified, resulting in a more abrupt drop in output.

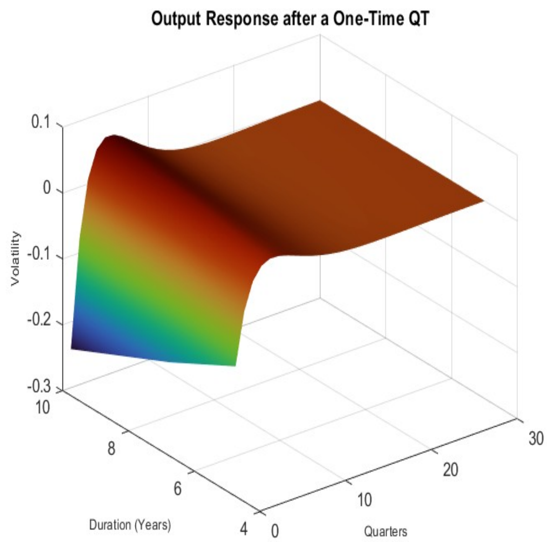

Figure 28: Role of Maturity of Government Debt

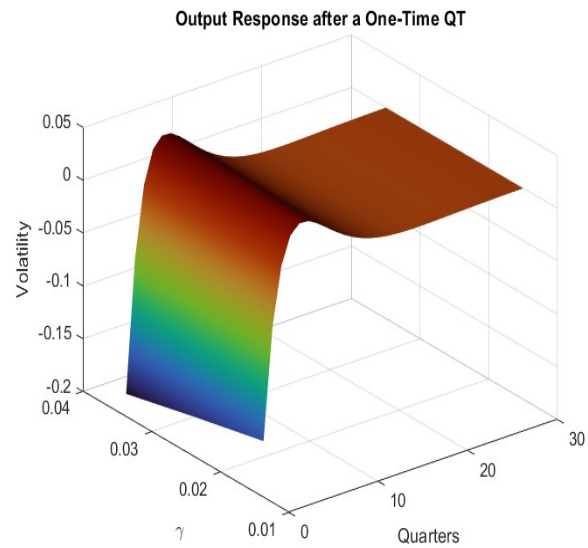

Figure 29: Role of Reserves

Next we compare the policies according to the monetary policy stance: how tight the policy rate should be in the QT process.

QT Strategy	Average Policy Rate
Discretion	2.22%
Commitment	1.69%
Lower Maturity: 1 Year	2.27%
Higher Reserves Liq. Benefits $\gamma = 0.05$	2.12%

Table 13: Monetary Policy Stance

6.1 The flattening of the Phillips Curve and the monetary policy normalization

We conduct an experiment to study how the interaction between the rate hike and QT change when the Phillips Curve is flatter: we increase the Rotemberg cost parameter 40%:

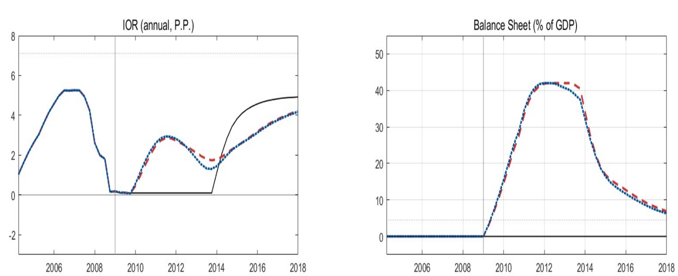

Figure 30: Interest Rate: Flatter PC. Discretion

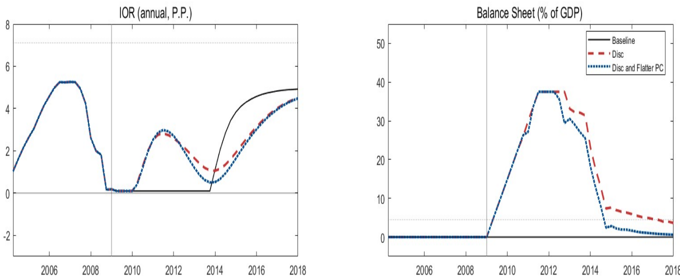

Figure 31: Interest Rate: Flatter PC. Commitment

With a flatter Phillips curve the interest rate is slightly higher than in the discretion benchmark case. When inflation starts converging to the target the decrease is more pronounced. The QT starts before in the scenario of a flatter Phillips Curve.

6.2 Summary

To summarize, Table 14 contrasts key stylized facts. The first columns present the policies implemented by major central banks, while the right column shows the optimal actions suggested by the model. The model's optimal policy aligns with the use of the policy rate as the primary tool for monetary policy conduction, and the lift-off occurring before the commencement of QT[11]. Additionally, under discretion, QT is more gradual compared to QE.

	FED/ECB/BOE Policy	**Constrained Optimal Policy**
Pace QE vs QT	QE aggressive and QT gradual	QE aggressive and QT gradual (Discretion)
Timing Rate Hikes vs QT	Rate Lift-Off and then QT	Rate Lift-Off and then QT
Active instrument away from ZLB	IOR	IOR

Table 14: Comparison of FED/ECB/BOE Policy and Constrained Optimal Policy

7 Related Literature

As Karadi and Nakov (2021) (henceforth NK) is a recent paper with a similar research question and theoretical framework, it's worth exploring the differences. First, they explore an occasionally binding constraint for the incentive constraint of the banks, their steady-state credit spreads are zero, and when the constraint is binding, they are positive. This is not the case in our framework, as we match different spreads from the data and the incentive constraint is always binding. Second, we model reserves explicitly, instead of QE being funded by deposits creation via households. Finally, our steady-state balance sheet is not

[11]Harrison (2024) finds the opposite result due to two key differences in our framework: first, the role of reserves' liquidity benefits, and second, our model incorporates capital.

zero. The investigation of announcement effects and the non-linear transitional dynamics are also only present in this paper.

Eggertson et al. (2023) develop a time consistent optimal policy where QE, reducing the maturity of government debt, is effective at the ZLB through a signaling channel: it generates expectations of a future monetary expansion. With commitment, as the Government can make use of forward guidance, the balance sheet plays no role. This is because there is no portfolio rebalancing channel in this model. Werning (2011) studies a standard three equation NK model with only the interest rate as instrument for the central bank, and in this setting, only a commitment optimal policy is effective. This policy is characterized by a temporary inflation boom, before the convergence to the target, a behaviour that is captured by the optimal policy projection.

Harrison (2024) studies a NK model with portfolio frictions between short term and long term bonds to study optimal QE and QT. He also compares a time-consistent with a commitment optimal policies. Optimal policies are characterized by aggressive QE and gradual QT, and QT starts before the interest rate lift off.

Cantore and Meichtry (2023) do not focus on optimal policy, but find though policy rules that interest rate lift-off before unwinding mitigates the negative effects of monetary policy normalization, a similar pattern that is obtained here.

8 Conclusion

Empirical evidence of Quantitative Tightening (QT) in the US indicates financial tightening and significant effects for Taper Tantrum and QT I announcements. The model demonstrates that announcing QT well in advance can mitigate its negative outcomes. Sales have short-term stimulative effects compared to unwinding, but the latter results in smoother effects once the policy is implemented. A strategy of announcing a passive unwinding and then surprising agents with aggressive government bonds sales results in high output volatility and lower welfare. Coordination policies that can dampen negative effects include liquidity facilities that provide stimulus while decreasing the stock of reserves and reducing the supply of government bonds.

A constrained optimal policy projection shows that gradual unwinding is optimal within a discretionary policy scenario. An optimal policy with commitment, leveraging forward guidance, requires less use of the balance sheet, allowing for a more aggressive QT than a time-consistent discretionary policy. If the demand for reserves is more sensitive/ features higher liquidity benefits, or the government debt maturity is higher, QT should be more gradual. This is also the case when there is finite planning, lower credibility and attention.

Future research can focus on studying the role of non-banks and the use of the Overnight Reverse Repo Facility. These two features characterized QT II, which primarily involved a reduction in the ONRRP while the liquidity drain via reserves remained slow.

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9 Appendix

9.1 Data

GDP: Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED

GDPDEF: Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted, FRED

CNP16OV: Civilian noninstitutional population, Thousands of Persons, Quarterly, Seasonally Adjusted, FRED

CNP16OV ma: A four-quarter trailing average of CNP16OV

PCEC: Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED

PCEDG: Personal Consumption Expenditures: Durable Goods, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED

GPDI: Gross Private Domestic Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate, FRED

AWHNONAG: Average Weekly Hours of Production and Nonsupervisory Employees: Total private, Hours, Quarterly, Seasonally Adjusted, FRED

CE16OV: Employment Level, Thousands of Persons, Quarterly, Seasonally Adjusted, FRED

COMPNFB: Nonfarm Business Sector: Compensation Per Hour, Index 2012=100, Quarterly, Seasonally Adjusted, FRED

FEDFUNDS: Effective Federal Funds Rate, Percent, FRED

GZ Spread: A corporate bond credit spread with high information content for economic activity constructed by Gilchrist and Zakrajsek (2012). Quarterly, Percent, Federal Reserve

Board.

FEDDT: Federal agency debt securities held by the Federal Reserve: All Maturities, Millions of Dollars, Quarterly, Not Seasonally Adjusted, FRED

Moody's Seasoned Aaa Corporate Bond Minus Federal Funds Rate (AAAFF)

9.1.1 National Financial Conditions Index: The Federal Reserve of Chicago

Credit Variables:

- 1-mo. Nonfinancial commercial paper A2P2/AA credit spread
- Markit Investment Grade (IG) 5-yr Senior CDS Index
- 30-yr Jumbo/Conforming fixed-rate mortgage spread
- Markit High Yield (HY) 5-yr Senior CDS Index
- BofA ML High Yield/Moody's Baa corporate bond yield spread
- CBOE Crude Oil Volatility Index, OVX
- NACM Survey of Credit Managers: Credit Manager's Index
- 30-yr Conforming Mortgage/10-yr Treasury yield spread
- Commercial Bank 24-mo. Personal Loan/2-yr Treasury yield spread
- Commercial Bank 48-mo. New Car Loan/2-yr Treasury yield spread
- UM Household Survey: Durable Goods Credit Conditions Good/Bad spread
- UM Household Survey: Mortgage Credit Conditions Good/Bad spread
- SP US Bankcard Credit Card: Excess Rate Spread
- NFIB Survey: Credit Harder to Get
- Bond Market Association Municipal Swap/State & Local Government 20-yr GO bond spread
- UM Household Survey: Auto Credit Conditions Good/Bad spread
- Moody's Baa corporate bond/10-yr Treasury yield spread

- SP US Bankcard Credit Card: 3-mo. Delinquency Rate
- FRB Senior Loan Officer Survey: Tightening Standards on Small C&I Loans
- FRB Senior Loan Officer Survey: Tightening Standards on RRE Loans
- FRB Senior Loan Officer Survey: Increasing spread on Small C&I Loans
- FRB Senior Loan Officer Survey: Increasing spread on Large C&I Loans
- FRB Senior Loan Officer Survey: Tightening Standards on CRE Loans
- FRB Senior Loan Officer Survey: Tightening Standards on Large C&I Loans
- NY Fed Consumer Credit Panel: Loan Delinquency Status: Non-current (Percent of Total Balance)
- Commercial Bank Noncurrent/Total Loans
- NY Fed Consumer Credit Panel: New Seriously Delinquent Loan Balances (Percent of Current Balance)
- NY Fed Consumer Credit Panel: New Delinquent Loan Balances (Percent of Current Balance)
- S&P US Bankcard Credit Card: Receivables Outstanding
- FRB Senior Loan Officer Survey: Willingness to Lend to Consumers
- Finance Company Owned & Managed Receivables
- Consumer Credit Outstanding
- MBA Serious Delinquencies
- Money Stock: MZM

Leverage Variables:

- S&P 500 Financials/S&P 500 Price Index (Relative to 2-yr MA)
- CME Eurodollar/CBOT T-Note Futures Market Depth
- S&P 500, S&P 500 mini, NASDAQ 100, NASDAQ mini Open Interest
- 3-mo. Eurodollar, 10-yr/3-mo. swap, 2-yr and 10-yr Treasury Open Interest

- Net Notional Value of Credit Derivatives
- CMBS Issuance (Relative to 12-mo. MA)
- Nonmortgage ABS Issuance (Relative to 12-mo. MA)
- New US Corporate Debt Issuance (Relative to 12-mo. MA)
- Commercial Bank Total Unused C&I Loan Commitments/Total Assets
- S&P 500, NASDAQ, and NYSE Market Capitalization/GDP
- 2-yr Constant Maturity Treasury yield
- New State & Local Government Debt Issues (Relative to 12-mo. hMA)
- Broker-dealer Debit Balances in Margin Accounts
- COMEX Gold/NYMEX WTI Futures Market Depth
- Commercial Bank Consumer Loans/Total Assets
- CoreLogic National House Price Index
- Commercial Bank C&I Loans/Total Assets
- Commercial Bank Securities in Bank Credit/Total Assets
- CME E-mini S&P Futures Market Depth
- Total Agency and GSE Assets/GDP
- New US Corporate Equity Issuance (Relative to 12-mo. MA)
- Total Assets of ABS issuers/GDP
- Wilshire 5000 Stock Price Index
- 10-yr Constant Maturity Treasury yield
- Household debt outstanding/PCE Durables and Residential Investment
- Total Assets of Insurance Companies/GDP
- Nonfinancial business debt outstanding/GDP
- Total Assets of Funding Corporations/GDP

- Total Assets of Broker-dealers/GDP
- Total Assets of Finance Companies/GDP
- Total REIT Assets/GDP
- Federal, state, and local debt outstanding/GDP
- Fed funds and Reverse Repurchase Agreements/Total Assets of Commercial Banks
- Total Assets of Pension Funds/GDP
- FRB Commercial Property Price Index
- Commercial Bank Real Estate Loans/Total Assets
- Total MBS Issuance (Relative to 12-mo. MA)

9.2 VAR Details

In the following plot, we present the responses of the 30-year Treasury yield and the 10-year term premium during QT I and QT II.

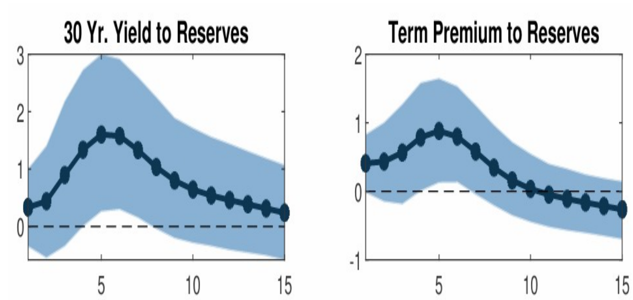


Figure 32: Oct 2017 - Sept 2019. (Spreads relative to IOR)

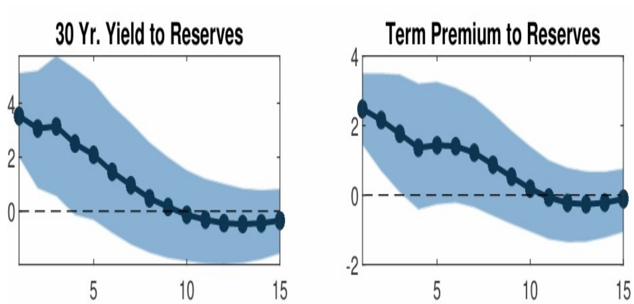


Figure 33: May 2022 - Feb 2024. (Spreads relative to IOR)

The response to a reduction in reserves during QT II, as seen in the Overnight Funding Borrowing cost, is:

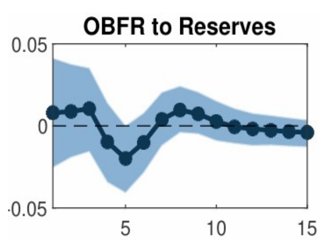


Figure 34: May 2022- Feb 2024. (Spreads wrt. IOR)

Table 15: Announcements Surrounding First Quantitative Tightening Episode[a]

Date	Announcement	Description
May 22, 2013[b]	Tapering	Tapering could begin “in the next few meetings”
Jun 19, 2013[c]	Tapering	Tapering could be appropriate “later this year”
Dec 18, 2013[c]	Tapering	FOMC announces it will start tapering its purchases of MBS and longer term treasuries of $35bn and $40bn per month
May 21, 2014[d]	Full Reinvestment	Minutes signal beginning of balance sheet normalization planning
Jul 9, 2014[d]	Full Reinvestment	Minutes discuss gradual approach to ceasing asset reinvestments
Jul 17, 2014[d]	Full Reinvestment	Further measured reductions in the pace of asset purchases might come in next meetings
Aug 20, 2014[d]	Full Reinvestment	Minutes offer details on balance sheet normalization planning
Sep 17, 2014[d]	Full Reinvestment	FOMC releases Policy Normalization Principles and Plan
Oct 29, 2014[d]	Full Reinvestment	FOMC announced the end of asset purchase program this month
Jan 12, 2017[e]	Full Reinvestment	Three Fed speeches discuss normalizing the balance sheet
Apr 5, 2017[d]	Full Reinvestment	Minutes signal phasing out reinvestments “later this year”
May 24, 2017[d]	Full Reinvestment	Minutes detail plan for phasing out reinvestment
Jun 14, 2017[c]	QT1	FOMC releases asset runoff plan, announces that runoff will begin “this year”
Sep. 20, 2017[c]	QT1	Announcement: asset runoff will begin next month
Jan 26, 2022[f]	QT2	Minutes issues “Principles for Reducing the Size of the Federal Reserve’s Balance Sheet”
Mar 16, 2022[f]	QT2	Reducing FED securities in a predictable manner
May 4, 2022[f]	QT2	FOMC adopts “Plans for Reducing the Size of the Federal Reserve’s Balance Sheet”
Sep 21, 2022[f]	QT2	Caps on Treasury securities and MBS redemptions double in September
Nov 2, 2022[f]	QT2	FOMC agrees to continue reducing the Federal Reserve’s securities holdings.

[a] Source: Smith and Valcarcel (2023) and Lu and Valcarcel (2023)
[b] Source: The Economic Outlook Congressional Hearings, 113th Congress, Joint Economic Committee.
`https://www.govinfo.gov/content/pkg/CHRG-113shrg81472/pdf/CHRG-113shrg81472.pdf`
[c] Source: FOMC Meeting Calendars, Statements, and Minutes (2016-2021).
`https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm`
[d] Source: Federal Reserve History of the FOMC’s Policy Normalization Discussions and Communications. `https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm`
[e] Source: Ben Bernanke’s Brookings Blog Shrinking the Fed’s Balance Sheet.
[f] Source: FOMC Meeting Calendars, Statements, and Minutes (2022).
`https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm`

9.3 Decomposing the yields during QT

In this section we follow closely Lloyd and Ostry (2024) to understand the impact of QT I in the ten year maturity bond under a Local-Projections approach. We decompose the yield into expectations of short term rates and a term premium.

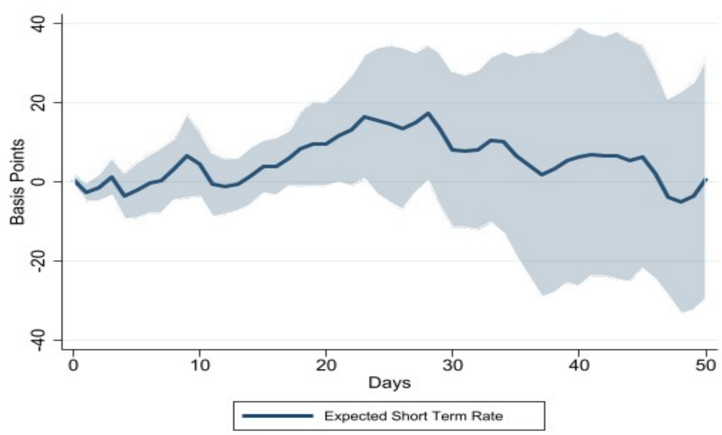
$$y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{lsap} + \delta_{QT}^h (\epsilon_t^{lsap} \mathcal{I}_t^{QT}) + \theta_{QT}^h \mathcal{I}_t^{QT} + \sigma^h x_t + u_t^h$$

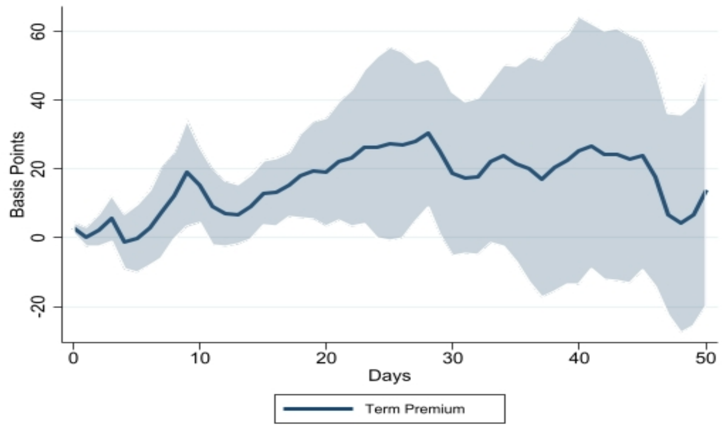
where x include the interest rate level to EFFR and forward guidance surprises from Swanson (2021) and the Bloomberg Economic Surprises to take into account both monetary policy and other economic events. 5 daily lags of the dependent variable are also included.

We decompose the expectations and the term premium, as follow:

$$y_t = exp_t + tp_t$$

where $exp_t = \frac{1}{10} \sum_{n=0}^{10-1} y_{1,t+n}^e$ and the term premium, as used in the SVAR is the series measured by Kim and Wright (2005)





60
40
20
0
-20
Basis Points
0
10
20
30
40
50
Days
Term Premium

9.4 The banks's problem

In this section we solve the financial intermediaries problem, extended with liquidity facilities.

During a QT process where reserves are scarcer, liquidity facilities work alleviating the credit constraint. We will assume that these facilities are also exogenous to the bank and provided at rate zero.

$$\Theta(M_t/N_t) = \frac{1}{\exp\Bigg(\theta\Bigg(1+\gamma\big(\frac{M_{j,t}+LQ_{j,t}}{N_{j,t}}\big)\Bigg)\Bigg)}$$

As in Boehl, Gavin, and Strobel (2022), Liquidity Facilities will follow an AR(1) process:

$$LQ_t = \rho_{LQ} LQ_{t-1} + \epsilon_t^{LQ}$$

The value of each bank is given by:

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty} (1-\sigma)\sigma^{j-1}\Lambda_{t,t+1} n_{j,t+1}$$

where $1-\sigma$ is the exit probability and n is the net worth.

Their balance sheet is:

$$Q_t^K S_t + Q_t^B B_t + M_t = D_t + LQ_t + N_t$$

The bank is subject to an incentive constraint as in Gertler and Kiyotaki (2010):

$$V_t \geq \Theta(M_t/N_t)\Bigg(Q_t^K S_t + \Delta^L Q_t^B B_t\Bigg)$$

The bank has balance-sheet costs. They are increasing in loans and decreasing in reserves.

$$\frac{dC(N_t,S_t)}{dS_t} = \kappa^L\Bigg(\max\left\{\frac{Q_t^K S_t}{N_t} - \frac{S}{N}, 0\right\}\Bigg)^2$$

We assume no absconding rates for reserves.

Liquidity injections relax the tightness of the constraint:

$$\Theta(M_t/N_t) = \frac{1}{\exp\left(\theta\big(1+\gamma(\frac{M_t+LQ_t}{N_t})\big)\right)}$$

$$\Theta'(M_t/N_t) = -(\frac{\theta\gamma}{N_t})\exp(^{-\theta(1+\frac{\gamma(M_t+LQ_t)}{N_t})})$$

$$N_{i,t} = (R^F_t - R^D_{t-1})Q^K_{t-1}S_{t-1} + (R^B_t - R^D_{t-1})Q^B_{t-1}B_{t-1} + (R^M_{t-1} - R^D_{t-1})M_{t-1}$$
$$+(R^D_{t-1} - R^{LQ}_{t-1})LQ_{t-1} + R^D_{t-1}N_{i,t-1} - C(S_{t-1}, N_{t-1})$$

The problem of the banks can be written recursively as:

$$V_{i,t} = (1-\sigma)\,\mathbb{E}_t\sum_{j=1}^{\infty}\sigma^{j-1}\Lambda_{t+j,t+j+1}\Big\{(R^F_{t+1+j} - R^d_{t+j})Q^K_{t+j}S_{t+j} + (R^B_{t+1+j} - R^d_{t+j})Q^B_{t+j}B_{t+j}$$
$$+(R^D_{t+j} - 1)LQ_{t+j} + (R^M_{t+j} - R^d_{t+j})M_{t+j} + R^d_{t+j}N_{t+j} - C(S_{t+j}, N_{t+j})\Big\}$$

The maximization problem is then:

$$\mathcal{L} = (1+\lambda_t)\Big[(1-\sigma)\,\mathbb{E}_t\,\Lambda_{t,t+1}n_{t+1} + \sigma\big[\mathbb{E}_t\,\Lambda_{t,t+1}V_{t+1}\big]\Big] - \lambda_t\Theta_t()(Q^K_t S_t + \Delta^L Q^B_t B_t)$$
$$\mathcal{L} = (1+\lambda_t)\Big[(1-\sigma)\,\mathbb{E}_t\,\Lambda_{t,t+1}\Big[(R^F_{t+1} - R^D_t)Q^K_t S_t + (R^B_{t+1} - R^D_t)Q^B_t B_t + (R^M_t - R^D_t)M_t +$$
$$+(R^D_t - R^{LQ}_t)LQ_{t-1} + R^D_t N_{i,t} - C(S_t, N_t)\Big] + \sigma\big[\mathbb{E}_t\,\Lambda_{t,t+1}V_{t+1}\big]\Big] - \lambda_t\Theta_t()(Q^K_t S_t + \Delta^L Q^B_t B_t)$$

The optimality conditions with respect to loans, bonds and reserves are:

$$\frac{d\mathcal{L}}{dS_t} = (1+\lambda_t)\Bigg\{ \mathbb{E}_t(1-\sigma)\Lambda_{t,t+1}\Big[(R^F_{t+1} - R^d_t - C'(S_t,N_t))\Big] +$$
$$\sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}\frac{dV_{t+1}}{dN_{t+1}}\Big[(R^F_{t+1} - R^d_t - C'(S_t,N_t))\Big]\Bigg\} = \lambda_t\Theta_t(M_t,N_t)$$

$$\frac{d\mathcal{L}}{dB_t} = (1+\lambda_t)\Bigg\{ \mathbb{E}_t(1-\sigma)\Lambda_{t,t+1}(R^B_{t+1} - R^d_t) + \sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}\frac{dV_{t+1}}{dN_{t+1}}(R^B_{t+1} - R^d_t)\Bigg\} = \lambda_t\Delta^L\Theta_t(M_t,N_t)$$

$$\frac{d\mathcal{L}}{dM_t} = (1+\lambda_t)\Bigg\{ \mathbb{E}_t(1-\sigma)\Lambda_{t,t+1}\big(R^M_t - R^d_t\big) + \sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}\frac{dV_{t+1}}{dN_{t+1}}\Big(R^M_t - R^d_t\Big)\Bigg\}$$
$$= \lambda_t\Theta'_t(M_t,N_t)(Q^K_t S_t + \Delta^L Q^B_t B_t)$$

Define $\Omega_{t+1} = 1-\sigma+\sigma\frac{dV_{t+1}}{dN_{t+1}}$ as the bankers augmented stochastic discount factor
So,

$$\mathbb{E}_t\,\Lambda_{t,t+1}\big(R^F_{t+1} - R^d_t - C'(S_t,N_t)\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Theta_t$$
$$\mathbb{E}_t\,\Lambda_{t,t+1}\big(R^B_{t+1} - R^d_t\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Delta^L\Theta_t$$
$$\mathbb{E}_t\,\Lambda_{t,t+1}\big(R^M_t - R^d_t\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Theta'_t(Q^K_t S_t + \Delta^L Q^B_t B_t)$$

The adjusted leverage is:

$$\phi_t = \frac{Q^K_t S_t + \Delta Q^B_t B_t}{N_t}$$

The value function, given a linear guess, can be written as:

$$V_t = (1-\sigma)\,\mathbb{E}_t\,\Lambda_{t,t+1}N_{t+1} + \sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}V_{t+1}$$
$$A_tN_t = (1-\sigma)\,\mathbb{E}_t\,\Lambda_{t,t+1}N_{t+1} + \sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}A_{t+1}N_{t+1}$$
$$A_tN_t = \mathbb{E}_t\,\Lambda_{t,t+1}N_{t+1}\Omega_{t+1}$$

$$\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}\Big[(R^F_{t+1} - R^D_t)Q^K_t S_t + (R^B_{t+1} - R^D_t)Q^B_t B_t$$
$$+(R^M_t - R^D_t)M_t + (R^D_t - R^{LQ}_t)LQ_t + R^D_t N_{i,t} - C(S_t, N_t)\Big]$$

So,

$$A_t N_t = \Lambda_{t,t+1}\Omega_{t,t+1}\Big[(R^F_{t+1} - R^D_t)Q^K_t S_t + (R^B_{t+1} - R^D_t)Q^B_t B_t$$
$$+(R^M_t - R^D_t)M_t + (R^D_t - R^{LQ}_t)LQ_t + R^D_t N_{i,t} - C(S_t, N_t)\Big]$$

$$\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}\Big[(R^F_{t+1} - R^D_t)Q^K_t S_t + (R^F_{t+1} - R^D_t)\Delta^L Q^B_t B_t$$
$$+\left(\frac{\Theta'_t}{\Theta_t}(R^F_{t+1} - R^D_t)(Q^K_t S_t + \Delta^L Q^B_t B_t)\right)M_t + (R^D_t - R^{LQ}_t)LQ_t + R^D_t N_{i,t} - C(S_t, N_t)\Big]$$
$$\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\Big[Q^K_t S_t + \Delta^L Q^B_t B_t$$
$$+\left(\frac{\Theta'_t}{\Theta_t}(Q^K_t S_t + \Delta^L Q^B_t B_t)\right)M_t\Big] + \Lambda_{t,t+1}\Omega_{t,t+1}R^D_t N_{i,t} + \Lambda_{t,t+1}\Omega_{t,t+1}(R^D_t - R^{LQ}_t)LQ_t - \Lambda_{t,t+1}\Omega_{t,t+1}C(S_t, N_t$$

Using the non-arbritrage condition between bonds and loans and the leverage definition:

$$\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\phi_t N_t\left[1 + \left(\frac{\Theta'_t}{\Theta_t}\right)M_t\right]$$
$$+\Lambda_{t,t+1}\Omega_{t,t+1}(R^D_t N_{i,t}) + \Lambda_{t,t+1}\Omega_{t,t+1}LQ_t(R^D_t - R^{LQ}_t) - \Lambda_{t,t+1}\Omega_{t,t+1}C(S_t, N_t)$$
$$\Lambda_{t,t+1}\Omega_{t,t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\phi_t\left[1 + \left(\frac{\Theta'_t}{\Theta_t}\right)M_t\right] + \Lambda_{t,t+1}\Omega_{t,t+1}(R^D_t + ((R^D_t - R^{LQ}_t)L\bar{Q}_t)$$
$$-\Lambda_{t,t+1}\Omega_{t,t+1}C(S_t, N_t)/N_t$$
$$\phi_t\Theta_t = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\phi_t\left[1 + \left(\frac{\Theta'_t}{\Theta_t}\right)M_t\right] + \Lambda_{t,t+1}\Omega_{t,t+1}(R^D_t + ((R^D_t - R^{LQ}_t)L\bar{Q}_t)$$
$$-\Lambda_{t,t+1}\Omega_{t,t+1}C(S_t, N_t)/N_t$$
$$\Theta_t = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\left[1 + \left(\frac{\Theta'_t}{\Theta_t}\right)M_t\right] + \frac{\Lambda_{t,t+1}\Omega_{t,t+1}(R^D_t + ((R^D_t - R^{LQ}_t)L\bar{Q}_t)}{\phi_t}$$
$$-\frac{\Lambda_{t,t+1}\Omega_{t,t+1}C(S_t, N_t)/N_t}{\phi_t}$$

So,

$$\phi_t = \frac{\mathbb{E}_t\,\Lambda_{t,t+1}\Omega_{t,t+1}\left(R^D_t + \Psi_t\right)}{\Theta_t(M_t/N_t) - \mathbb{E}_t\,\Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\left[1 + \left(\frac{M_t\Theta'_t()}{\Theta_t()}\right)\right]}$$

where $L\bar{Q}_t$ is liquidity injection normalized by net worth. We will assume zero interest rate for this policy tool. The funding costs are: $\Psi_t = \Lambda_{t,t+1}\Omega_{t,t+1}((R^D_t - R^{LQ}_t)L\bar{Q}_t)$
$- \Lambda_{t,t+1}\Omega_{t,t+1}C(S_t, N_t)/N_t$

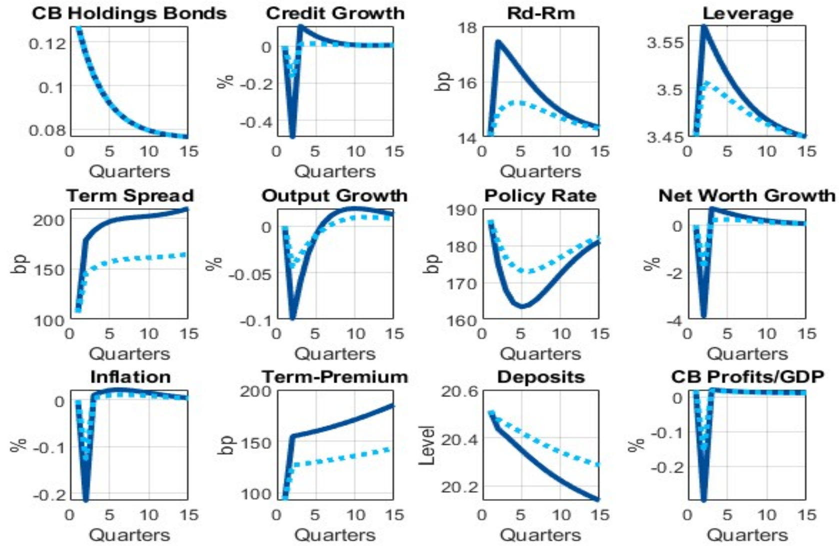


Figure 35: Dynamics after a BS reduction. The blue line is the benchmark scenario. The light blue dotted line is the scenario with the central bank liquidity facilities at 1.5% of GDP. We calibrate to match the average amounts of the sum of Central Bank Liquidity Swaps, Net Portfolio Holdings of Commercial Paper Funding Facility LLC , Term auction credit and Other loans during the last quarter of 2008 and the last quarter of 2019.

9.5 The Role of Slow-Moving Capital: Bank Equity

The value of each bank is given by:

$$V_{j,t} = \mathbb{E}_t \sum_{j=1}^{\infty} (1-\sigma)\sigma^{j-1}\Lambda_{t,t+1} n_{j,t+1}$$

where $1-\sigma$ is the exit probability and n is the net worth.

Their balance sheet is:

$$Q_t^K S_t + Q_t^B B_t + M_t = D_t + LQ_t + N_t$$

The bank is subject to an incentive constraint as in Gertler and Kiyotaki (2010):

$$V_t \geq \Theta(M_t/N_t)\Big(Q_t^K S_t + \Delta^L Q_t^B B_t\Big)$$

The bank has balance-sheet costs. They are increasing in loans and decreasing in reserves.

$$C_t(e_t, N_t) = \frac{\kappa}{2}\frac{e_t^2}{N_t}$$

We assume no absconding rates for reserves.

$$\begin{aligned} N_{i,t} = (R_t^F - R_{t-1}^D)Q_{t-1}^K S_{t-1} + (R_t^B - R_{t-1}^D)Q_{t-1}^B B_{t-1} + (R_{t-1}^M - R_{t-1}^D)M_{t-1} \\ +(R_{t-1}^D - R_{t-1}^{LQ})LQ_{t-1} + R_{t-1}^D N_{i,t-1} + e_{t-1} \end{aligned}$$

$$\mathcal{L} = (1+\lambda_t)\Big[(1-\sigma)\,\mathbb{E}_t\,\Lambda_{t,t+1}n_{t+1} + \sigma\big[\,\mathbb{E}_t\,\Lambda_{t,t+1}V_{t+1} - e_t - C_t(e_t,N_t)\big]\Big] - \lambda_t\Theta_t()(Q_t^K S_t + \Delta^L Q_t^B B_t)$$

$$\begin{aligned} \mathcal{L} = (1+\lambda_t)\Big[(1-\sigma)\,\mathbb{E}_t\,\Lambda_{t,t+1}\Big[(R_{t+1}^F - R_t^D)Q_t^K S_t + (R_{t+1}^B - R_t^D)Q_t^B B_t + (R_t^M - R_t^D)M_t + \\ +(R_t^D - R_t^{LQ})LQ_{t-1} + R_t^D N_{i,t} + e_t\Big] + \sigma\big[\,\mathbb{E}_t\,\Lambda_{t,t+1}V_{t+1} - e_t - C_t(e_t,N_t)\big]\Big] - \lambda_t\Theta_t()(Q_t^K S_t + \Delta^L Q_t^B B_t) \end{aligned}$$

$$\frac{d\mathcal{L}}{dS_t} = (1+\lambda_t)\Bigg\{\mathbb{E}_t(1-\sigma)\Lambda_{t,t+1}\Big[(R^F_{t+1} - R^d_t)\Big] + \sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}\frac{dV_{t+1}}{dN_{t+1}}\Big[(R^K_{t+1} - R^d_t)\Big]\Bigg\} = \lambda_t\Theta_t()$$

$$\frac{d\mathcal{L}}{dB_t} = (1+\lambda_t)\Bigg\{\mathbb{E}_t(1-\sigma)\Lambda_{t,t+1}(R^B_{t+1} - R^d_t) + \sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}\frac{dV_{t+1}}{dN_{t+1}}(R^B_{t+1} - R^d_t)\Bigg\} = \lambda_t\Delta^L\Theta_t()$$

$$\frac{d\mathcal{L}}{dM_t} = (1+\lambda_t)\Bigg\{\mathbb{E}_t(1-\sigma)\Lambda_{t,t+1}\big(R^M_t - R^d_t\big) + \sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}\frac{dV_{t+1}}{dN_{t+1}}\Big(R^M_t - R^d_t\Big)\Bigg\}$$
$$= \lambda_t\Theta_t()'(Q^K_t S_t + \Delta^L Q^B_t B_t)$$

Define $\Omega_{t+1} = 1 - \sigma + \sigma\frac{dV_{t+1}}{dN_{t+1}}$ as the bankers augmented stochastic discount factor
So,

$$\sigma\Lambda_{t,t+1}\left[\frac{dV_{t+1}}{dN_{t+1}} - 1 - \kappa\frac{e_t}{N_t}\right] = 0$$
$$\mathbb{E}_t\,\Lambda_{t,t+1}\big(R^F_{t+1} - R^d_t\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Theta_t()$$
$$\mathbb{E}_t\,\Lambda_{t,t+1}\big(R^B_{t+1} - R^d_t\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Delta^L\Theta_t()$$
$$\mathbb{E}_t\,\Lambda_{t,t+1}\big(R^M_t - R^d_t\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Theta_t()'(Q^K_t S_t + \Delta^L Q^B_t B_t)$$

The Equity FOC is:

$$\xi_t = \frac{\mathbb{E}_t\,\Lambda_{t,t+1}(\Omega_{t+1} - 1)}{\sigma\kappa}$$

The adjusted leverage is:

$$\phi_t = \frac{Q^K_t S_t + \Delta Q^B_t B_t}{N_t}$$

$$V_t = (1-\sigma)\,\mathbb{E}_t\,\Lambda_{t,t+1}N_{t+1} + \sigma\Big[\mathbb{E}_t\,\Lambda_{t,t+1}V_{t+1} - e_t - C_t(e_t, N_t)\Big]$$
$$A_t N_t = (1-\sigma)\,\mathbb{E}_t\,\Lambda_{t,t+1}N_{t+1} + \sigma\,\mathbb{E}_t\,\Lambda_{t,t+1}A_{t+1}N_{t+1}$$
$$A_t N_t = \mathbb{E}_t\,\Lambda_{t,t+1}N_{t+1}\Omega_{t+1}$$

$$\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}\Big[(R^F_{t+1} - R^D_t)Q^K_t S_t + (R^B_{t+1} - R^D_t)Q^B_t B_t$$
$$+(R^M_t - R^D_t)M_t + (R^D_t - R^{LQ}_t)LQ_t + R^D_t N_{i,t} - C_t(e_t, N_t)\Big]$$

So,

$$A_t N_t = \Lambda_{t,t+1}\Omega_{t,t+1}\Big[(R^F_{t+1} - R^D_t)Q^K_t S_t + (R^B_{t+1} - R^D_t)Q^B_t B_t$$
$$+(R^M_t - R^D_t)M_t + R^D_t N_{i,t} - C_t(e_t, N_t)\Big]$$

$$\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}\Big[(R^F_{t+1} - R^D_t)Q^S_t S_t + (R^F_{t+1} - R^D_t +)\Delta^L Q^B_t B_t$$
$$+\left(\frac{\Theta_t()'}{\Theta_t()}(R^F_{t+1} - R^D_t)(Q^K_t S_t + \Delta^L Q^B_t B_t)\right)M_t + (R^D_t - R^{LQ}_t)LQ_t + R^D_t N_{i,t} - C_t(e_t, N_t)\Big]$$
$$\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\Big[Q^K_t S_t + \Delta^L Q^B_t B_t$$
$$+\left(\frac{\Theta_t()'}{\Theta_t()}(Q^K_t S_t + \Delta^L Q^B_t B_t)\right)M_t\Big] - +\Lambda_{t,t+1}\Omega_{t,t+1}R^D_t N_{i,t} + (R^D_t - R^{LQ}_t)LQ_t - C_t(e_t, N_t)$$
$$\Lambda_{t,t+1}\Omega_{t,t+1}N_{t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\phi_t N_t\left[1 + \left(\frac{\Theta_t()'}{\Theta_t()}\right)M_t\right]$$
$$+\Lambda_{t,t+1}\Omega_{t,t+1}(R^D_t N_{i,t}) + \Lambda_{t,t+1}\Omega_{t,t+1}LQ_t(R^D_t - R^{LQ}_t) - \Lambda_{t,t+1}\Omega_{t,t+1}C_t(e_t, N_t)$$
$$\Lambda_{t,t+1}\Omega_{t,t+1} = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\phi_t\left[1 + \left(\frac{\Theta_t()'}{\Theta_t()}\right)M_t\right] + \Lambda_{t,t+1}\Omega_{t,t+1}(R^D_t + ((R^D_t - R^{LQ}_t)\bar{LQ}_t) -$$
$$\Lambda_{t,t+1}\Omega_{t,t+1}C_t(e_t, N_t)/N_t$$
$$\phi_t\theta_t = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\phi_t\left[1 + \left(\frac{\Theta_t()'}{\Theta_t()}\right)M_t\right] + \Lambda_{t,t+1}\Omega_{t,t+1}(R^D_t + ((R^D_t - R^{LQ}_t)\bar{LQ}_t) -$$
$$\Lambda_{t,t+1}\Omega_{t,t+1}C_t(e_t, N_t)/N_t$$
$$\Theta_t() = \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\left[1 + \left(\frac{\Theta_t()'}{\Theta_t()}\right)M_t\right] + \Lambda_{t,t+1}\Omega_{t,t+1}\frac{X_t}{\phi_t}$$

So,

$$\bar{\phi}_t = \frac{\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t,t+1} \Big(X_t \Big)}{\Theta_t(M_t/N_t) - \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t,t+1} (R^F_{t+1} - R^D_t) \Big[1 + \Big(\frac{M_t \Theta'_t()}{\Theta_t()} \Big) \Big]}$$

where $L\bar{Q}_t$ is liquidity injection normalized by net worth. We will assume zero interest rate for this policy tool. The variable $X_t = (R^D_t + ((R^D_t - R^{LQ}_t) L\bar{Q}_t) - C_t(e_t, N_t)/N_t$

$$\Theta(M_t/N_t) = \frac{1}{\exp\Big(\theta \big(1 + \gamma (\frac{M_t + LQ_t}{N_t}) \big) \Big)}$$

$$\Theta'(M_t/N_t) = -(\frac{\theta \gamma}{N_t}) \exp(^{-\theta(1 + \frac{\gamma(M_t + LQ_t)}{N_t})})$$

Using as in Karadi and Nakov (2021) $\kappa = 28$ we get almost similar dynamics than in our baseline model with quadratic leverage costs:

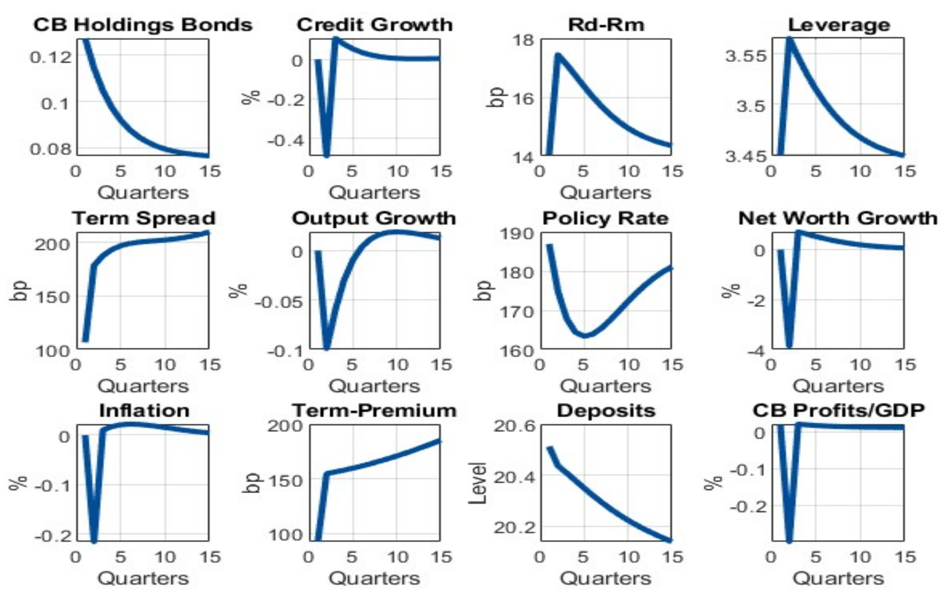


Figure 36: Impact on Macro Variables

9.6 Bank Mechanisms

$$N_{j,t} = (R^F_t - R^D_{t-1})Q^K_{t-1}S_{j,t-1} + (R^B_t - R^D_{t-1})Q^B_{t-1}B_{j,t-1} + (R^M_{t-1} - R^D_{t-1})M_{j,t-1}$$
$$+R^D_{t-1}N_{j,t-1} - C(N_{j,t-1}, Q^K_{t-1}S_{j,t-1})$$

The bank profits are defined as $\Pi^B_t = \frac{\Pi_t N_t}{N_{t-1}}$

$$\Pi^B_t = \Pi_t(R^F_t - R^D_{t-1})\frac{Q^K_{t-1}S_{t-1}}{N_{t-1}} + \Pi_t(R^B_t - R^D_{t-1})\frac{Q^B_{t-1}B_{t-1}}{N_{t-1}} + \Pi_t(R^M_{t-1} - R^D_{t-1})\frac{M_{t-1}}{N_{t-1}}$$
$$+R^D_{t-1} - C(N_{t-1}, Q^K_{t-1}S_{t-1})$$

$$\Pi^B_t = \Pi_t R^F_t \frac{Q^K_{t-1}S_{t-1}}{N_{t-1}} + \Pi_t R^B_t \frac{Q^B_{t-1}B_{t-1}}{N_{t-1}} + \Pi_t(R^M_{t-1} - R^D_{t-1})\frac{M_{t-1}}{N_{t-1}}$$
$$-\Pi_t R^D_{t-1}\frac{Q^B_{t-1}B_{t-1}}{N_{t-1}} - \Pi_t R^D_{t-1}\frac{Q^K_{t-1}S_{t-1}}{N_{t-1}} + \Pi_t R^D_{t-1} - \Pi_t C(N_{t-1}, Q^K_{t-1}S_{t-1})$$

Adding and substracting $\mathbb{E}_{t-1}\,\Pi_t R^F_t \frac{Q^K_{t-1}S_{t-1}}{N_{t-1}}$ and $E_{t-1}\Pi_t R^B_t \frac{Q^B_{t-1}B_{t-1}}{N_{t-1}}$:

$$\Pi^B_t = \Pi_t R^F_t \frac{Q^K_{t-1}S_{t-1}}{N_{t-1}} + \Pi_t R^B_t \frac{Q^B_{t-1}B_{t-1}}{N_{t-1}} + \Pi_t(R^M_{t-1} - R^D_{t-1})\frac{M_{t-1}}{N_{t-1}}$$
$$-\Pi_t R^D_{t-1}\frac{Q^B_{t-1}B_{t-1}}{N_{t-1}} - \Pi_t R^D_{t-1}\frac{Q^K_{t-1}S_{t-1}}{N_{t-1}} + \Pi_t R^D_{t-1} - \Pi_t C(N_{t-1}, Q^K_{t-1}S_{t-1})$$
$$+\mathbb{E}_{t-1}\,\Pi_t R^F_t \frac{Q^K_{t-1}S_{t-1}}{N_{t-1}} - \mathbb{E}_{t-1}\,\Pi_t R^F_t \frac{Q^K_{t-1}S_{t-1}}{N_{t-1}}$$
$$+\mathbb{E}_{t-1}\,\Pi_t R^B_t \frac{Q^B_{t-1}B_{t-1}}{N_{t-1}} - \mathbb{E}_{t-1}\,\Pi_t R^B_t \frac{Q^B_{t-1}B_{t-1}}{N_{t-1}}$$

$$\Pi^B_t = \left(\Pi_t R^F_t - \mathbb{E}_{t-1}\,\Pi_t R^F_t\right)\frac{Q^K_{t-1}S_{t-1}}{N_{t-1}} + \left(\Pi_t R^B_t - \mathbb{E}_{t-1}\,\Pi_t R^B_t\right)\frac{Q^B_{t-1}B_{t-1}}{N_{t-1}} + (R^M_{t-1} - R^D_{t-1})\frac{M_{t-1}}{N_{t-1}}$$
$$+\left(\mathbb{E}_{t-1}\,\Pi_t R^F_t - R^D_{t-1}\right)\frac{Q^K_{t-1}S_{t-1}}{N_{t-1}} + \left(\mathbb{E}_{t-1}\,\Pi_t R^B_t - R^D_{t-1}\right)\frac{Q^B_{t-1}B_{t-1}}{N_{t-1}} + R^D_{t-1} - C(N_{t-1}, Q^K_{t-1}S_{t-1})$$

Defining , $\Phi^K_t = \frac{Q^K_t S_t}{N_t}$, $\Phi^M_t = \frac{M_t}{N_t}$ and $\Phi^B_t = \frac{Q^B_t B_t}{N_t}$, $CS_t = \mathbb{E}_t\,\Pi_{t+1}R^F_{t+1} - R^D_t$, $TP_t = \mathbb{E}_t\,\Pi_{t+1}R^B_{t+1} - R^D_t$ and log-linearizing around the steady-state:

$$\hat{\pi}^B_t = \frac{R^F\Phi^K}{\Pi^B}\big(\hat{\pi}_t - \mathbb{E}_{t-1}\,\hat{\pi}_t\big) + \frac{mpk\Phi^K}{\Pi^B}\big(\hat{mpk}_t - \mathbb{E}_{t-1}\,\hat{mpk}_t\big) + \frac{\Phi^K}{\pi^B}\big(\hat{q}_t - \mathbb{E}_{t-1}\,\hat{q}_t\big) + \frac{CS\Phi^K}{\Pi^B}\hat{cs}_{t-1}$$
$$\frac{R^B\Phi^B}{\Pi^B}\big(\hat{\pi}_t - \mathbb{E}_{t-1}\,\hat{\pi}_t\big) + \frac{\Phi^B}{\pi^B}\big(\hat{q}^B_t - \mathbb{E}_{t-1}\,\hat{q}^B_t\big) + \frac{TP\Phi^B}{\Pi^B}\hat{tp}_{t-1}$$
$$\frac{R^M\Phi^M}{\Pi^B}\hat{r}^M_{t-1} - \frac{R^D\Phi^M}{\Pi^B}\hat{r}^D_{t-1} + \frac{R^D}{\Pi^B}\hat{r}^D_{t-1} - \frac{C}{\Pi^B}\hat{c}_{t-1}$$

We can separate this effect between surprises/announcements and implementation:

$$\begin{aligned}\hat{\pi}^B_t = {} & \underbrace{\frac{R^F\Phi^K + R^B\Phi^B}{\Pi^B}\big(\hat{\pi}_t - \mathbb{E}_{t-1}\,\hat{\pi}_t\big)}_{\text{Announcement: Inflation}} + \underbrace{\frac{mpk\Phi^K}{\Pi^B}\big(\hat{mpk}_t - \mathbb{E}_{t-1}\,\hat{mpk}_t\big)}_{\text{Announcement: Dividend}} \\ & + \underbrace{\frac{\Phi^K}{\Pi^B}\big(\hat{q}^k_t - \mathbb{E}_{t-1}\,\hat{q}^k_t\big) + \frac{\Phi^B}{\Pi^B}\big(\hat{q}^B_t - \mathbb{E}_{t-1}\,\hat{q}^B_t\big)}_{\text{Announcement: Capital Gains}} + \underbrace{\frac{CS\Phi^K}{\Pi^B}\hat{cs}_{t-1}}_{\text{Credit Spreads}} + \underbrace{\frac{TS\Phi^B}{\Pi^B}\hat{ts}_{t-1}}_{\text{Term Spread}} \\ & + \underbrace{\frac{R^M\Phi^M}{\Pi^B}\hat{r}^M_{t-1} - \frac{R^D\Phi^M}{\Pi^B}\hat{r}^D_{t-1}}_{\text{Short Term Rates Difference}} + \underbrace{\frac{R^D}{\Pi^B}\hat{r}^D_{t-1}}_{\text{Deposit Rate}} - \underbrace{\frac{C}{\Pi^B}\hat{c}_{t-1}}_{\text{Leverage Cost}}\end{aligned}$$

9.7 Announcement Effects

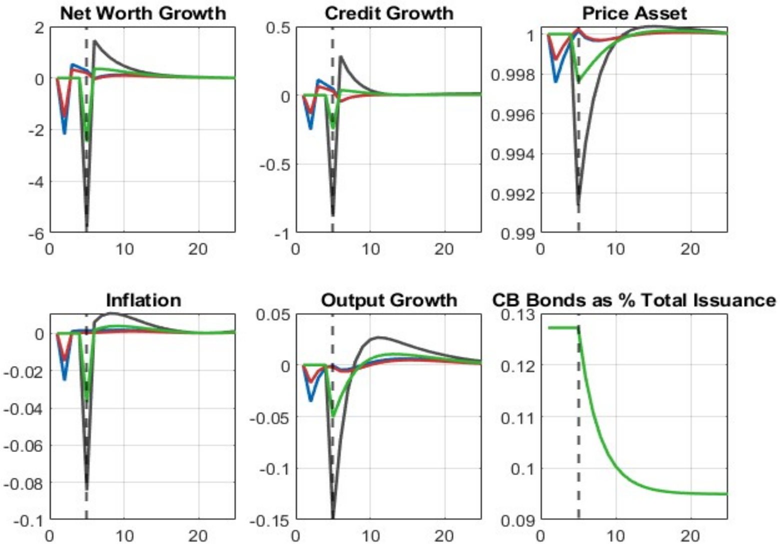


Figure 37: Announcements: The blue line represents the announcement made 4 quarters before the start of the policy, while the black dashed line reflects the outcome of the announcement with only 1 quarter of anticipation. The red line represents the announcement made 4 quarters before the start of the policy without leverage costs and when $\gamma \to 0$, while the green line reflects the outcome of the announcement with only 1 quarter of anticipation and $\gamma \to 0$.

9.8 Crisis: QE and Unexpected One-Time Sell Off

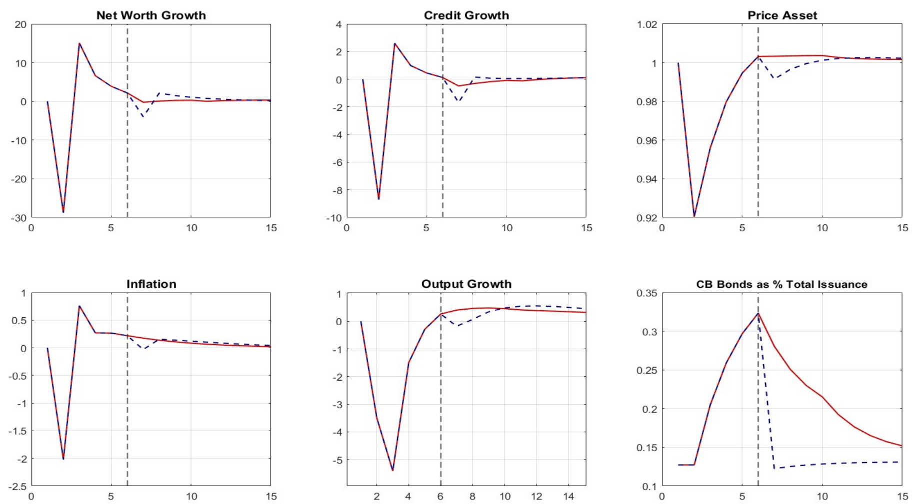


Figure 38: Crisis Event: Unexpected One-Time Sell-Off vs Passive Unwinding. The blue dashed lines plot the sales scenario while the red ones the passive unwinding one.

9.9 Reserve Demand

In this section we follow closely De Groot and Haas (2023) to develop a simple model to capture the liquidity benefits of reserves and its relation to a cost function that behaves as an LCR requirement. We also impose a leverage constraint. A bank starts with loans L and government bonds B and $D_r = L+B$ retail deposits. The bank also takes wholesale deposits, $D_w = M$ and places them at the central bank to obtain M reserves. At t=1 loans, reserves and bonds are repaid, as both types of deposits. After that a fraction $D = \sigma(D_r + D_w)$ of total deposits are withdrawn of the bank. The cost function captures interbank market frictions and illiquidity of bonds. The cost is decreasing holding liquid assets, that are reserves and bonds. The latter have a haircut/lower liquidity than reserves.

The cost function is:

$$C(D, M, B) = \frac{2\theta}{1+\epsilon}\Bigg(\max\big(D - M - \psi B, 0\big)\Bigg)^{1+\epsilon}$$

where $\epsilon > 1$ and $\sigma, \theta \in (0,1)$

The leverage constraint is : $L \leq \phi(L + \Delta B)$

$$\max_{M,B,L}\Bigg\{(R^L - R^D)L + (R^B - R^D)B + (R^M - R^D)M - \frac{2\theta}{1+\epsilon}\Bigg(\max(\sigma(L+B+M) - M - \psi B, 0)\Bigg)^{1+\epsilon}\Bigg\}$$
$$+\lambda\Bigg(L(\phi - 1) + \phi\Delta B\Bigg)$$

The optimality conditions are:

$$(R^L - R^D) - \theta\Bigg(\max(\sigma(L+B+M) - M - \psi B, 0)\Bigg)^{\epsilon}\sigma + \lambda(\phi - 1) = 0$$

$$(R^B - R^D) - \theta\Bigg(\max(\sigma(L+B+M) - M - \psi B, 0)\Bigg)^{\epsilon}(\sigma - \psi) + \lambda\phi\Delta = 0$$

$$(R^M - R^D) - \theta\Bigg(\max(\sigma(L+B+A) - A - \psi B, 0)\Bigg)^{\epsilon}(\sigma - 1) = 0$$

The LM is higher the higher is the liquidity risk ,σ , the iliquidity of loans θ, the tighter is the bank's constraint ϕ and the lower are the spreads (bank profits).

$$\lambda_t = \frac{\theta\sigma\bigg(\max(\sigma(L+B+M)-M-\psi B,0)\bigg)^{\epsilon} - (R^L - R^D)}{\phi - 1}$$

When we combine the optimality conditions, the relation between the spreads is :

$$R^B - R^D = (R^M - R^D)\left[\frac{\sigma}{\sigma-1} - \frac{\psi}{\sigma-1} - \frac{\sigma\phi\Delta}{(\sigma-1)(\phi-1)}\right] + (R^L - R^D)\left[\frac{\phi\Delta}{\phi-1}\right]$$

The spread between government bonds and deposits is a weighted average between the convenience yield that depends on the liquidity premium of reserves over bonds and the loan spread, adjusted by the liquidity premium of bonds over loans, Δ.

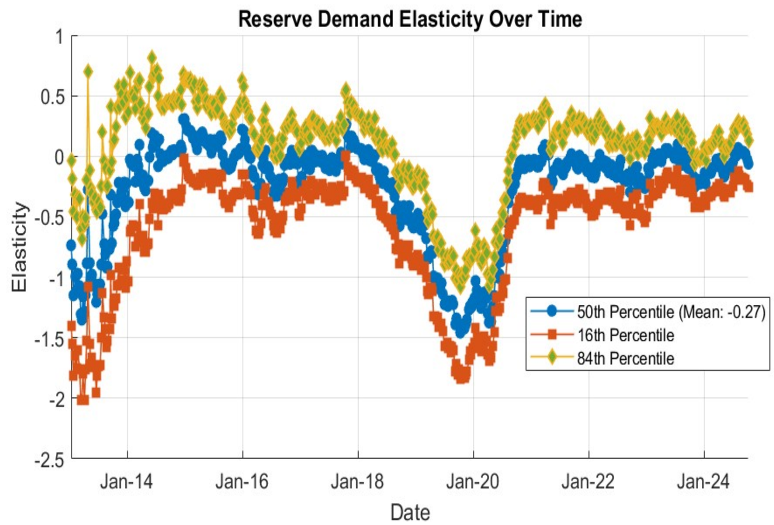
The optimal level of reserves:

$$M^* = \frac{1}{\sigma-1}\left(\frac{R^M - R^D}{\theta(\sigma-1)}\right)^{\frac{1}{\epsilon}} - \frac{\sigma}{\sigma-1}L - \frac{(\sigma-\psi)}{\sigma-1}B$$

$$M^* = \frac{\sigma}{1-\sigma}L + \frac{(\sigma-\psi)}{1-\sigma}B - \frac{1}{1-\sigma}\left(\frac{R^D - R^M}{\theta(1-\sigma)}\right)^{\frac{1}{\epsilon}}$$

Reserves are increasing in loans and the illiquidity of them, θ, as reserves decrease the holding costs. When the government bonds are liquid enough such that $\psi > \sigma$, reserves are decreasing in the holdings of them.

9.10 NY FED: Reserve Demand Elasticity



9.11 Log-Linearization

Market Clearing for loans/capital and bonds

$$b_t = b_t^G \frac{B^G}{B} - b_t^h \frac{B^H}{B} - b_t^{CB} \frac{B^{CB}}{B}$$
$$b_t^H = \frac{(r_{t+1}^b - r_t^d)}{\kappa \bar{B}^H} \frac{R^B}{R^D}$$

From $Q_t^K K_t = Q_t^S S_t$ and $\phi_t = \frac{Q_t^K K_t + \Delta^L Q_t^B B_t}{N_t}$

$$k_t = -\frac{\Delta Q^B B}{Q^K S} b_t - \frac{\Delta Q^B B}{Q^K S} q_t^b + \frac{\phi N}{Q^K S}(\phi + n_t) - Q_t^K$$
$$k_t = -\frac{\Delta Q^B B}{Q^K S}\left[b_t^G \frac{B^G}{B} - b_t^h \frac{B^H}{B} - b_t^{CB} \frac{B^{CB}}{B}\right] - \frac{\Delta Q^B B}{Q^K S} q_t^b + \frac{\phi N}{Q^K S}(\phi_t + n_t) - q_t^K$$

Using the household demand for government debt, $\kappa B^h b_t^h = \frac{R^b}{R^d}(r_{t+1}^b - r_t^d)$ and $q_t^b = \frac{R^B(r_t^b + q_{t-1})}{\gamma^b}$

$$k_t = s_t = \underbrace{b_t^{CB} \frac{\Delta Q^B B^{CB}}{S}}_{\text{Direct QE Channel}} + \underbrace{\left[\frac{1}{\kappa B^H} \frac{R^B}{R^D}(r_{t+1}^b - r_t^d)\right] \frac{\Delta Q^B B^H}{S}}_{\text{Household Debt Demand}} - \underbrace{b_t^G \frac{\Delta Q^B B^G}{S}}_{\text{Debt Supply}} - \underbrace{\frac{\Delta Q^B B}{S}\left(\frac{R^B(r_t^b + q_{t-1})}{\gamma^b}\right)}_{\text{Bond Yield Effect}}$$
$$+ \underbrace{\frac{\phi N}{S}(\phi_t + n_t)}_{\text{Leverage and Net Worth effect}} - \underbrace{q_t^K}_{\text{Capital Price}}$$

The maximum adjusted leverage:

$$\hat{\phi}_t = \frac{\nu_t^d}{\Theta\left(\frac{M_t}{N_t}\right) - \nu_t^k(1 + \Psi_t)}$$

where $\nu_{k,t} = \mathbb{E}_t \Lambda_{t,t+1}\Omega_{t,t+1}(R_{t+1}^K - R_t^d)$ is the credit spread and $\nu_{d,t} = \mathbb{E}_t \Lambda_{t,t+1}\Omega_{t,t+1} R_t^d$ is the funding cost.

$$\Omega\Omega_{t+1} = \phi\Theta(M/N)(\phi_t + \Theta_t(M_t/N_t)) - \nu^k \phi(\phi_t + \nu_t^k) - \phi\nu^k\Psi(\phi_t + \nu_t^k + \Psi_t)$$

Or,

$$\Omega\Omega_{t+1} - \phi\Theta(M/N)\Theta_t(M_t/N_t) + \phi\nu^k(\nu_t^k + \nu_t^k\Psi + \Psi\Psi_t) = \phi\phi_t\Big(\Theta(M/N) - \nu^k(1+\Psi)\Big)$$

So:

$$\phi_t = \frac{\Omega\Omega_{t+1} - \phi\Theta(M/N)\Theta_t(M_t/N_t) + \phi\nu^k(\nu_t^k + \nu_t^k\Psi + \Psi\Psi_t)}{\phi\Theta(M/N) - \phi\nu^k(1+\Psi)}$$

Net-Worth:

$$Nn_t = \sigma\Big\{(R^F - R^D)K(q_{t-1}^k + k_{t-1}) + R^F K r_t^F + (R^B - R^D)Q^B B(q_{t-1}^b + b_{t-1}) + Q^B B R^B r_t^b$$
$$+(R^M - R^D)M(m_{t-1}) + MR^M r_t^m - R^D D r_{t-1}^d + R^d N n_{t-1}\Big\} + X$$

Using $b_t = b_t^G \frac{B^G}{B} - b_t^h \frac{B^H}{B} - b_t^{CB}\frac{B^{CB}}{B}$, $m_t = q_t^b + b_t^{CB}$and $M = Q^B B^{CB}$

$$n_{t+1} = \frac{\sigma}{N}\Big\{(R^F - R^D)K(q_t^k + k_t) + R^F K r_{t+1}^F + (R^B - R^D)Q^B B\Big(q_t^b + b_t^G\frac{B^G}{B} - b_t^h\frac{B^H}{B} - b_t^{CB}\frac{B^{CB}}{B}\Big)$$
$$+Q^B B R^B r_{t+1}^b + (R^M - R^D)Q^B B^{CB}(q_t^b + b_t^{CB}) + Q^B B^{CB} R^M r_{t+1}^m - R^D D r_t^d\Big\} + \sigma R^d n_t + X$$

The linearized augmented stochastic discount factor is:

$$\Omega\Omega_t = \Omega r_{t-1}^d + \sigma\big(\nu_n \nu_{n,t} + \nu_n\big)$$

9.12 Leverage Constraint

$$V_{j,t} = \Theta\left(\frac{M_{j,t}}{N_{j,t}}\right)\Big(Q_t^K S_{j,t} + \Delta^L Q_t^B B_{j,t}^B\Big)$$

$$V_t = \nu_t^k Q_t^K S_t + \nu_t^b Q_t^B B_t + \nu_t^m M_t + \nu_t^n N_t$$

Using $\phi_t = \frac{Q_t^K S_t + \Delta Q_t^B B_t}{N_t}$, then $V = \Theta(M_t/N_t)N_t\phi_t$. We also use from the FOCs that $\nu_t^b = \nu_t^k\Delta$

$$\phi_t = \frac{\nu_t^k \phi_t + \nu_t^m M_t + \nu_t^n N_t}{\Theta(M_t/N_t)}$$

Defining $m_t = \frac{M_t}{N_t}$:

$$\phi_t = \frac{\nu_t^n + \nu_t^m m_t}{\Theta(N_t/N_t) - \nu_t^k}$$

The growth rate of the net-worth and adjusted leverage is:

$$z_t = \frac{N_{t+1}}{N_t} = \frac{(R_{t+1}^F - R_t^d)\phi_t N_t + R_t^d N_t + (R_t^M - R_t^d)M_t}{N_t} = (R_{t+1}^F - R_t^d)\phi_t + R_t^d + (R_t^M - R_t^d)m_t$$

And,

$$x_{t+1}\frac{Q_{t+1}^K S_{t+1} + \Delta Q_{t+1}^B B_{t+1}}{Q_t^K S_t + \Delta Q_t^B B_t} = \frac{\phi_{t+1} N_{t+1}}{\phi_t N_t} = \frac{\phi_{t+1}}{\phi_t} z_{t+1}$$

where:

$$\nu_t^k = \mathbb{E}_t \left\{ (1-\sigma)\Lambda_{t,t+1}(R_{t+1}^F - R_t^d) + \sigma \Lambda_{t,t+1} x_{t+1} \nu_{t+1}^k \right\}$$

and

$$\nu_t^n = \mathbb{E}_t \left\{ (1-\sigma) + \sigma \Lambda_{t,t+1} z_{t+1} \nu_{t+1}^n \right\}$$

$$\nu^k \nu_t^k = \mathbb{E}_t \left\{ (1-\sigma)\frac{R^F}{R^d}(R_{t+1}^F - R_t^d) + \sigma \nu^k \beta z (x_{t+1} + \nu_{t+1}^k) \right\}$$

$$\nu_t^n = \sigma z \beta \, \mathbb{E}_t \left\{ (z_{t+1} + \nu_{t+1}^n) \right\}$$

$$x_t = \phi_t - \phi_{t-1} + z_t$$

We write the growth of net-worth as: $z_t / R_t^d = \frac{(R_{t+1}^F - R_t^d)\phi_t}{R_t^d} + \frac{(R_t^M - R_t^d)}{R^d} m_t + 1$. In steady-state: $z\beta = \frac{(R^F - R^d)\phi + (R^M - R^d)m}{R^d} + 1$

$$z_t = \frac{\phi\frac{R^F}{R^d}(R^F_{t+1} - R^d_t) + (\frac{R^F}{R^d} - 1)\phi\phi_t + m\frac{R^m}{R^d}(R^m_t - R^d_t) + (\frac{R^M}{R^d} - 1)mm_t}{(\frac{R^F}{R^d} - 1)\phi + (\frac{R^M}{R^d} - 1)M + 1}$$
$$= \frac{\phi\frac{R^F}{R^d}(R^F_{t+1} - R^d_t) + (\frac{R^F}{R^d} - 1)\phi\phi_t + m\frac{R^m}{R^d}(R^m_t - R^d_t) + (\frac{R^M}{R^d} - 1)mm_t}{\beta z}$$

$$\nu^k\nu^k_t = \mathbb{E}_t\Bigg\{(1-\sigma)\frac{R^F}{R^d}(R^F_{t+1} - R^d_t) +$$
$$\sigma\nu^k\beta z\Bigg(\Bigg[\phi_{t+1} - \phi_t + \frac{\phi\frac{R^F}{R^d}(R^F_{t+1} - R^d_t) + (\frac{R^F}{R^d} - 1)\phi\phi_t + m\frac{R^m}{R^d}(R^m_t - R^d_t) + (\frac{R^M}{R^d} - 1)mm_t}{\beta z}\Bigg] + \nu^k_{t+1}\Bigg)\Bigg\}$$

After some algebra:

$$\nu^k\nu^k_t = \mathbb{E}_t\Bigg\{(1-\sigma+\sigma\phi\nu^k)\frac{R^F}{R^d}(R^F_{t+1} - R^d) + \sigma\beta\nu^k z(\phi_{t+1} + \nu_{t+1}) + \sigma\nu^k L_t + \phi_t\sigma\nu^k\Bigg[-\beta z + \phi(\frac{R^F}{R^d} - 1)\Bigg]\Bigg\}$$

Now,

$$\nu^n_t = \sigma z\beta\,\mathbb{E}_t\Bigg\{(z_{t+1} + \nu^n_{t+1})\Bigg\}$$

$$\nu^n_t = \sigma\phi\,\mathbb{E}_t\Bigg\{\frac{R^F}{R^d}(R^F_{t+1} - R^d_t) + (\frac{R^F}{R^d} - 1)\phi_t + L_t + \frac{z\beta}{\phi}\nu^n_{t+1}\Bigg\}$$

We know that:

$$\phi_t = \frac{\nu^n\nu^n_t + L_t}{\phi(\Theta - \nu^k)} + \frac{\nu^k\nu^k_t - \Theta\Theta_t}{\Theta - \nu^k}$$

$$\phi_t = \frac{\sigma\nu^n\,\mathbb{E}_t\left\{\frac{R^F}{R^d}(R^F_{t+1}-R^d_t)+(\frac{R^F}{R^d}-1)\phi_t+L_t+\frac{z\beta}{\phi}\nu^n_{t+1}\right\}}{(\Theta-\nu^k)}$$

$$+\frac{\mathbb{E}_t\left\{(1-\sigma+\sigma\phi\nu^k)\frac{R^F}{R^d}(R^F_{t+1}-R^d)+\sigma\beta\nu^k z(\phi_{t+1}+\nu^k_{t+1})+\sigma\nu^k L_t+\phi_t\sigma\nu^k\Big[-\beta z+\phi(\frac{R^F}{R^d}-1)\Big]\right\}}{(\Theta-\nu^k)}$$

$$+\frac{L_t}{\phi(\Theta-\nu^k)}-\frac{\Theta\Theta_t}{\Theta-\nu^k}$$

Using: $\phi_{t+1}=\frac{\nu^n\nu^n_{t+1}}{\phi(\Theta-\nu^k)}+\frac{\nu^k\nu^k_{t+1}}{\Theta-\nu^k}+\frac{L_{t+1}}{\phi(\Theta-\nu^k)}-\frac{\Theta\Theta_t}{\Theta-\nu^k}$

$$\phi_t = \frac{\sigma\nu^n\,\mathbb{E}_t\left\{\frac{R^F}{R^d}(R^F_{t+1}-R^d_t)+(\frac{R^F}{R^d}-1)\phi_t+L_t\right\}}{(\Theta-\nu^k)}+\sigma\beta z\Big[\phi_{t+1}-\frac{L_{t+1}}{\phi(\Theta-\nu^k)}+\frac{\Theta\Theta_{t+1}}{\Theta-\nu^k}\Big]$$

$$+\frac{\mathbb{E}_t\left\{(1-\sigma+\sigma\phi\nu^k)\frac{R^F}{R^d}(R^F_{t+1}-R^d)+\sigma\beta\nu^k z\phi_{t+1}+\sigma\nu^k L_t+\phi_t\sigma\nu^k\Big[-\beta z+\phi(\frac{R^F}{R^d}-1)\Big]\right\}}{(\Theta-\nu^k)}$$

$$+\frac{L_t}{\phi(\Theta-\nu^k)}-\frac{\Theta\Theta_t}{\Theta-\nu^k}$$

Using: $\nu^n=\frac{1-\sigma}{1-\sigma\beta z}$ and $\nu^k=\frac{(1-\sigma)\beta(\frac{R^F}{R^d}-1)}{1-\sigma\beta z}$ so $\frac{\nu^k}{\nu^n}=\beta(\frac{R^F}{R^d}-1)$

$$\phi_t = \frac{\sigma\nu^n\,\mathbb{E}_t\left\{\frac{R^F}{R^d}(R^F_{t+1}-R^d_t)+(\frac{R^F}{R^d}-1)\phi_t+L_t\right\}}{(\Theta-\nu^k)}+\sigma\beta z\Big[\phi_{t+1}-\frac{L_{t+1}}{\phi(\Theta-\nu^k)}+\frac{\Theta\Theta_{t+1}}{\Theta-\nu^k}\Big]$$

$$+\frac{\mathbb{E}_t\left\{(1-\sigma+\sigma\phi\nu^k)\frac{R^F}{R^d}(R^F_{t+1}-R^d)+\sigma\beta\nu^k z\phi_{t+1}+\sigma\nu^k L_t+\phi_t\sigma\nu^k\Big[-\beta z+\phi(\frac{R^F}{R^d}-1)\Big]\right\}}{(\Theta-\nu^k)}$$

$$+\frac{L_t}{\phi(\Theta-\nu^k)}-\frac{\Theta\Theta_t}{\Theta-\nu^k}$$

$$\phi_t = \frac{\sigma\nu^n\,\mathbb{E}_t\left(\frac{R^F}{R^d}(R^F_{t+1}-R^d_t)+(\frac{R^F}{R^d}-1)\phi_t\right)+(1-\sigma+\sigma\phi\nu^k)\frac{R^F}{R^d}(R^F_{t+1}-R^d)+\phi_t\sigma\nu^k\Big[-\beta z+\phi(\frac{R^F}{R^d}-1)\Big]}{\Theta-\nu^k}$$

$$+\sigma\beta z\phi_{t+1}\left(1+\frac{\nu^k}{\Theta-\nu^k}\right)+\frac{\sigma\beta z\Theta\Theta_{t+1}-\Theta_t\Theta}{\Theta-\nu^k}+\frac{(\sigma(\nu^n+\nu^k)+1)L_t}{\Theta-\nu^k}-\frac{\sigma\beta z L_{t+1}}{\phi(\Theta-\nu^k)}$$

$$\phi_t = \frac{\sigma\nu^n\,\mathbb{E}_t\left(\frac{R^F}{R^d}(R^F_{t+1}-R^d_t)+(\frac{R^F}{R^d}-1)\phi_t\right)+(1-\sigma+\sigma\phi\nu^k)\frac{R^F}{R^d}(R^F_{t+1}-R^d)+\phi_t\sigma\nu^k\Big[-\beta z+\phi(\frac{R^F}{R^d}-1)\Big]}{\Theta-\nu^k}$$

$$+\sigma\beta z\phi_{t+1}\left(1+\frac{\nu^k}{\Theta-\nu^k}\right)+\frac{\sigma\beta z\Theta\Theta_{t+1}-\Theta_t\Theta}{\Theta-\nu^k}+\frac{(\sigma(\nu^n+\nu^k)+1)L_t}{\Theta-\nu^k}-\frac{\sigma\beta zL_{t+1}}{\phi(\Theta-\nu^k)}$$

$$\phi_t = \frac{\frac{R^F}{R^d}(R^F_{t+1}-R^d_t)(1-\sigma\sigma\phi\nu^k+\sigma\nu^n)}{\Theta-\nu^k}+\frac{\phi_t\sigma\nu^k\left(1/\beta-\beta z+\phi(\frac{R^F}{R^d}-1)\right)}{\Theta-\nu^k}$$

$$+\sigma\beta z\phi_{t+1}\left(1+\frac{\nu^k}{\Theta-\nu^k}\right)+\frac{\sigma\beta z\Theta\Theta_{t+1}-\Theta_t\Theta}{\Theta-\nu^k}+\frac{(\sigma(\nu^n+\nu^k)+1)L_t}{\Theta-\nu^k}-\frac{\sigma\beta zL_{t+1}}{\phi(\Theta-\nu^k)}$$

9.13 A Toy Model

In this section, we develop a simple framework to illustrate how QT tightens the financial variables. The model follows Anderson and Cesa-Bianchi (2024) and Gertler and Karadi (2011) There are two agents: a continuum of firms and a financial intermediary. The firms finance their capital expenditures via a bank loan (S) at an interest rate (R^S). The bank invest in firm loans, reserves and government bonds. As in GKK economies, it's subject to an incentive constraint, where the tightness is a decreasing function of reserves, not a parameter. The liabilities of the bank are their own net worth (N^B) and deposits.

9.13.1 Firms

$$\max_{K} zK^{\alpha} - R^S S$$

$$st.$$

$$K = S$$

After solving, the credit spread can be written as:

$$\frac{R^S}{R} = CS = \frac{\alpha z K^{\alpha-1}}{R}$$

9.13.2 Banks

$$\max_{D,S,B,M} R^S S + R^M M + R^B B - R(S + B + M - N^B)$$

$$R^S S + R^M M + R^B B - R(S + B + M - N^B) \geq \Theta(M_t)\big(S + \Delta B\big)$$

$$S + B + M = N^B + D$$

The optimization problem can be written as:

$$L = R^S S + R^M M + R^B B - R(S + B + M - N^B) + \lambda\Big\{R^S S + R^M M + R^B B - R(S + B + M - N^B) - \Theta(M_t)(S + \Delta B)\Big\}$$

$$\lambda = \frac{R - R^S}{R^S - R - \Theta(M)}$$
$$\lambda = \frac{R - R^B}{R^B - R - \Delta\Theta(M)}$$
$$\lambda = \frac{R - R^M}{R^M - R - \Theta(M)'(S + \Delta B)}$$

FOC LM:

$$\left\{ R^S S + R^M M + R^B B - R(S + B + M - N^B) - \Theta(M_t)\big(S + \Delta B\big) \right\}$$

Using $CS = \frac{R^S}{R}$, $L = M(\hat{R}^M - 1) + B(\hat{R}^B - 1)$ as liquidity holdings, $\phi = \frac{S+\Delta B}{N^B}$ as leverage and $S = K$:

$$CSS + \hat{R}^M M + \hat{R}^B B - (S + B + M - N^B) - \frac{\Theta(M)}{R}\big(S + \Delta B\big) = 0$$

Solving for the credit spread:

$$CS = \frac{\frac{\Theta(M)N^B\phi}{R} + S - L - N^B}{S}$$

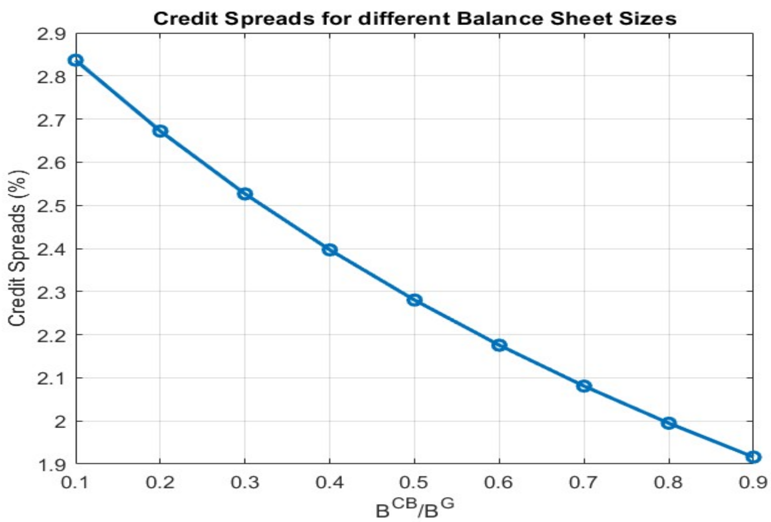
Credit supply schedule is a piece-wise function with a kink where the financial constraint becomes binding. The credit spread is increasing on the tightness parameter of the incentive constraint $\Theta(M)$, increasing on the total capital demand by the firms, and decreasing in liquid assets holdings and total net worth (sum of the firms and bankers net-worth). When the incentive constraint does no bind, $\lambda = 0$ and the credit supply schedule as well as the government bonds and reserves spreads can be recovered from the first order conditions.

QT affects the credit supply schedule as the financial intermediaries absorb bonds and reserves are taken out of the market: it increases the tightness of the constraint, it decreases the liquid asset holdings and increases leverage. ($\phi = \frac{S+\Delta B}{N^B}$)

$$CS = \begin{cases} 1 & \text{if } K < \frac{R(L+N^B)}{\Theta(M)} - \Delta B \\ \frac{\frac{\Theta(M)N^B\phi}{R} + S - L - N^B}{S} & \text{if } o.w \end{cases} \tag{24}$$

Using $M = B^{CB}$ and $B = B^G - B^{CB}$

$$CS = \begin{cases} 1 & \text{if } K < \frac{R(L+N^B)}{\Theta(M)} - \Delta(B^G - M) \\ \frac{\frac{\Theta(M)N^B\phi}{R}+S-L-N^B}{S} & \text{if } o.w \end{cases} \tag{25}$$



9.14 Solution Concept and Computation

The dynamics of the reduction of the balance sheet and the crisis simulations focus on the full nonlinear solution as our model features nonlinearities and nonmonotonicities. The method is detailed on this appendix.

9.14.1 Perfect Foresight Simulations

We use a two-boundary problem to solve the model non-linearly. The model can be written as:

$$f(y_{t+1}, y_t, y_{t-1}, u_t) = 0$$

We are interested in a trajectory Y: values for $y_1, ...y_T$ given shocks and $y_0, ...y_T$. Particularly, we solve a stacked system of equations: $f(y_2, y_1, y_0, u_1) = 0$ until $f(y_{T+1}, y_T, y_{T-1}, u_T = 0)$). for initial condition y_0 and a terminal condition y_{T+1}

The algorithm consists on guessing an initial value for Y, the steady-state in our case, and then updating using a Newton algorithm. Updated solutions are obtained by solving:

$$(Y^{k+1} - Y^k) = -J^{-1}F(Y^k)$$

until $||(F(Y^k)|| < \epsilon$

where J is the Jacobian matrix. For computation we use sparse Jacobians with the help of sparse matrix algebra libraries.

For the one-time shock IRFs we use standard perturbation methods.

9.15 Equations

Households

The functional form of the adjustment costs are given by:

$$\Phi_t^B = \frac{1}{2}\kappa\big(B_t - \bar{B}\big)^2$$

$$\mathbb{E}_t \,\Lambda_{t,t+1} R_t^D \Pi_{t+1}^{-1} = 1 \tag{26}$$

$$\Lambda_{t,t+1} = \beta \frac{U_C(C_{t+1}, L_{t+1})}{U_C(C_t, L_t)} \tag{27}$$

$$\Lambda_{t-1,t} = \beta \frac{\mu_t}{\mu_{t-1}}$$

$$W_t = -\frac{U_L(C_t, L_t)}{U_C(C_{t+1}, L_{t+1})} \tag{28}$$

$$B_t = \overline{B} + \frac{\mathbb{E}_t \,\Lambda_{t+1}(R_{t+1}^B - R_t^d)}{\kappa}$$

$$R_{t+1}^B = \mathbb{E}_t \,\frac{c + \gamma^b Q_{t+1}^B}{Q_t^B}$$

Banks

$$R_{t+1}^F = \frac{R_{t+1}^K + (1-\delta)Q_{t+1}^K}{Q_t^K}\xi_{t+1}$$

$$\Omega_{t+1} = 1 - \sigma + \sigma\Theta_t\phi_t$$

$$Q_t^K S_{j,t} + Q_t^B B_{j,t}^B + M_{j,t} = D_{j,t} + N_{j,t}$$

$$\mathbb{E}_t \, \Lambda_{t,t+1}\big(R^F_{t+1} - R^d_t - \kappa^S\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Theta_t$$

$$\mathbb{E}_t \, \Lambda_{t,t+1}\big(R^B_{t+1} - R^d_t\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Delta^L\Theta_t$$

$$\mathbb{E}_t \, \Lambda_{t,t+1}\big(R^M_t - R^d_t\big)\Omega_{t+1} = \frac{\lambda_t}{1+\lambda_t}\Theta'_t(Q^K_t S_t + \Delta^L Q^B_t B_t)$$

$$N_t = \sigma\Bigg[(R^F_t - R^D_{t-1})Q^K_{t-1}S_{t-1} + (R^B_t - R^D_{t-1})Q^B_{t-1}B_{t-1} + (R^M_{t-1} - R^D_{t-1})M_{t-1} \\ + R^D_{t-1}N_{t-1} - C\Bigg(S_{t-1}, N_{t-1}\Bigg)\Bigg] + X$$

$$\Theta(M_t/N_t) = \frac{1}{\exp\Bigg(\theta(1+\gamma(\frac{M_{j,t}}{N_{j,t}}))\Bigg)}$$

$$\phi_t = \frac{Q^K_t S_t + \Delta Q^B_t B_t}{N_t}$$

$$\bar{\phi}_t = \frac{\mathbb{E}_t \, \Lambda_{t,t+1}\Omega_{t,t+1}\Bigg(R^D_t - C_t\Bigg)}{\Theta_t(M_t/N_t) - \mathbb{E}_t \, \Lambda_{t,t+1}\Omega_{t,t+1}(R^F_{t+1} - R^D_t)\Bigg[1 + \Psi_t\Bigg]}$$

Firms

$$Y_t(i) = Z_t(K_t(i))^\alpha (L_t(i))^{1-\alpha}$$

$$K_{t+1} = \xi_{t+1}\big[I_t + (1-\delta)K_t\big]$$

$$Q^K_t = 1 + \phi_k \frac{I_j}{I_{j-1}}\Bigg(\frac{I_j}{I_{j-1}} - 1\Bigg) + \frac{\phi_k}{2}\Bigg(\frac{I_j}{I_{j-1}} - 1\Bigg)^2 - \phi_k \Lambda_{t,j}\frac{I^2_{j+1}}{I^2_j}\Bigg(\frac{I_j}{I_{j-1}} - 1\Bigg)$$

$$1-\epsilon+\epsilon mc_t=\phi^P(\pi_t-\pi)\pi_t-\phi^P\,\mathbb{E}_t\left[M_{t,t+1}\frac{y_{t+1}}{y_t}\pi_{t+1}^2(\pi_{t+1}-\pi)\right]$$
$$mc_t=\left(\frac{1}{\alpha}\right)^{\alpha}\left(\frac{1}{1-\alpha}\right)^{1-\alpha}\frac{1}{z}w_t^{1-\alpha}(R_t^k)^{\alpha}$$

Policy

$$R_t^M=\max\left\{\left(R_{t-1}^M\right)^{\rho_i}\left[R^M\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\rho_i}\epsilon_t^M,1\right\}$$

$$M_t=Q_t^B B_t^{CB}$$
$$B_t^{CB}=\min\left\{(1-\rho_{CB})B^{CB}+\rho_{CB}B_{t-1}^{CB}+\sum_{j\geq 0}^{T}\epsilon_{t|t-j}^{CB},\overline{B}^{CBmax}\right\}$$

$$\Pi_t^{CB}=B_{t-1}^{CB}Q_{t-1}^B R_t^B-Q_t^B B_t^{L,CB}+M_t-R_{t-1}^M M_{t-1}-\tau(Q_{t-1}^B B_{t-1}^{CB})^2$$

$$T_t+B_t^G+\Pi_t^{CB}=G_t+Q_{t-1}^B B_{t-1}^G R_t^B$$

$$B_t^G=(1-\rho_{B^G})B^G+\rho_{B^G}B_{t-1}^G+\epsilon_t^{B^G}$$
$$G_t=(1-\rho_g)G+\rho_g G_{t-1}+\epsilon_t^G$$

Market Clearing and Resource Constraint

$$B_t^G=B_t^H+B_t+B_t^{CB}$$

$$S_t=K_t$$

$$Y_t=C_t+I_t(1+s(\frac{I_t}{I_{t-1}}))+G_t+\frac{\kappa^P}{2}(\Pi_t-\Pi)^2Y_t+\tau Q_t^B B_t^{CB}$$

9.16 Two-Period Model

The households first order conditions with respect to deposits and bonds are:

$$\beta \Lambda R^D = 1$$

$$\frac{\beta \Lambda}{Q^B + \kappa^H (B^H - \bar{B}^H)} = 1$$

The loan return, given full depreciation, is:

$$R^F = \alpha \xi K^{\alpha - 1} - 1$$

We use the three optimality conditions of the financial intermediaries:

$$\beta \Lambda (R^F - R^D) = \Theta(M/N) \frac{\lambda}{1 + \lambda}$$
$$\beta \Lambda (R^B - R^D) = \Delta^L \Theta(M/N) \frac{\lambda}{1 + \lambda}$$
$$\beta \Lambda (R^M - R^D) = \frac{\lambda}{1 + \lambda} \Theta'(M/N)(S + \Delta Q^B B)$$

The market clearing conditions of loans/capital and bonds are:

$$K = S$$

$$B^G = B + B^H + B^{CB}$$

We solve for the LM of the banks problem, and then differentiate with respect to reserves:

$$\lambda = \frac{\beta \Lambda (R^F - R^D)}{\Theta(M/N) - \beta \Lambda (R^F - R^D)}$$

$$\frac{d\lambda}{dM} = \frac{\left\{ \left[\beta \Lambda (R^F - R^D)\right]' \left[\Theta(M/N) - \beta \Lambda (R^F - R^D)\right] - \left[\beta \Lambda (R^F - R^D)\right] \left[\Theta(M/N) - \beta \Lambda (R^F - R^D)\right]' \right\}}{\left[\Theta(M/N) - \beta \Lambda (R^F - R^D)\right]^2}$$

$$\frac{d\lambda}{dM} = (1 + \lambda)^2 \frac{\beta \Lambda (R^F - R^D)'}{\Theta(M/N)}$$

The numerator is:

$$\beta\Lambda(R^F - R^D)' = \beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\frac{dK}{dM} - \beta\Lambda\frac{dR^D}{dM}$$

Now we differentiate with respect to reserve the quantity of government bonds:

$$\frac{dB^H}{dM} = -(1/\kappa^H)\frac{dQ^B}{dM}$$

$$\underbrace{\frac{dB^G}{dM}}_{=0} = \frac{dB}{dM} + \frac{dB^H}{dM} + \underbrace{\frac{dB^{CB}}{dM}}_{1/Q^B}$$

Using $R^B = \frac{1}{Q^B}$ and $R^B - R^D = \Delta(R^F - R^D)$

$$\frac{1}{Q^B} - R^D = \Delta\Big(\alpha K^{\alpha-1} - 1 - R^D\Big)$$

So,

$$\frac{dQ^B}{dM} = \underbrace{-\Delta\alpha(\alpha-1)K^{\alpha-2}Q^{B2}}_{(+)}\frac{dK}{dM} - \underbrace{\Bigg(1-\Delta\Bigg)Q^{B2}}_{(-)}\frac{dR^D}{dM}$$

Differentiation of the incentive constraint:

$$N\frac{d\lambda}{dM}+(1+\lambda)B_{-1}\frac{Q^B}{dM} = \Theta(M/N)'K+\Theta(M/N)\frac{dK}{dM}+\Theta(M/N)'\Delta Q^B B+\Theta(M/N)\Delta\Bigg(Q^B\frac{dB}{dM}+B\frac{dQ^B}{dM}\Bigg)$$

$$N\frac{d\lambda}{dM} + (1+\lambda)B_{-1}\frac{Q^B}{dM} = \Theta(M/N)'K + \Theta(M/N)\frac{dK}{dM}$$
$$+\Theta(M/N)'\Delta Q^B B + \Theta(M/N)\Delta\Bigg(Q^B \underbrace{\frac{dB}{dM}}_{(\frac{1}{\kappa^H}-\frac{1}{Q^B})\frac{dQ^B}{dM}} + B\frac{dQ^B}{dM}\Bigg)$$

And using

$$\frac{d\lambda}{dM} = \frac{(1+\lambda)^2}{\Theta(M/N)}\Bigg(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\frac{dK}{dM} - \beta\Lambda\frac{dR^D}{dM}\Bigg)$$

Assuming $\frac{dR^D}{dM} = 0$ and solving for the impact of reserves on credit:

$$\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\frac{dK}{dM}\right)\right]+(1+\lambda)B_{-1}\frac{Q^B}{dM}=\Theta(M/N)'K+\Theta(M/N)\frac{dK}{dM}$$
$$+\Theta(M/N)'\Delta Q^B B+\Theta(M/N)\Delta\left(Q^B\underbrace{\frac{dB}{dM}}_{(\frac{1}{\kappa^H}-\frac{1}{Q^B})\frac{dQ^B}{dM}}+B\frac{dQ^B}{dM}\right)$$

$$\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\right)\right]\frac{dK}{dM}-\Theta(M/N)\frac{dK}{dM}=\Theta(M/N)'K$$
$$+\Theta(M/N)'\Delta Q^B B+\Theta(M/N)\Delta\frac{dQ^B}{dM}\left(\frac{Q^B}{\kappa^H}-1+B\right)-(1+\lambda)B_{-1}\frac{dQ^B}{dM}$$

$$\frac{dK}{dM}\left\{\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\right)\right]-\Theta(M/N)\right\}=\Theta(M/N)'\big(K+\Delta Q^B B\big)$$
$$+\frac{dQ^B}{dM}\left\{\Theta(M/N)\Delta\left(\frac{Q^B}{\kappa^H}-1+B\right)-(1+\lambda)B_{-1}\right\}$$

$$\frac{dK}{dM}=\frac{\Theta(M/N)'\big(K+\Delta Q^B B\big)}{\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\right)\right]-\Theta(M/N)}$$
$$+\frac{\frac{dQ^B}{dM}\left\{\Theta(M/N)\Delta\left(\frac{Q^B}{\kappa^H}-1+B\right)-(1+\lambda)B_{-1}\right\}}{\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\right)\right]-\Theta(M/N)}$$

$$\frac{dK}{dM}=\frac{\Theta(M/N)'\big(K+\Delta Q^B B\big)}{\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\right)\right]-\Theta(M/N)}$$
$$+\frac{\left\{\Theta(M/N)\Delta\left(\frac{Q^B}{\kappa^H}-1+B\right)-(1+\lambda)B_{-1}\right\}}{\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\right)\right]-\Theta(M/N)}(-\Delta\alpha(\alpha-1)K^{\alpha-2}Q^{B^2})\frac{dK}{dM}$$

$$\frac{dK}{dM} = \frac{A}{C} + \frac{DE}{C}\frac{dK}{dM}$$
$$\frac{dK}{dM} = \frac{A}{C - DE}$$

As $\Theta'(M/N) < 0$, then $A < 0$, $C < 0$ as the term in parenthesis is negative due to $\alpha < 1$ and $\Theta(M/N)$ is positive. $E > 0$ for the same reason. We need to prove that $C - DE < 0$:

$$\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\right)\right] - \Theta(M/N)$$
$$-\Theta(M/N)\Delta\left(\frac{Q^B}{\kappa^H} - 1 + B\right)\left(-\Delta\alpha(\alpha-1)K^{\alpha-2}Q^{B^2}\right)$$
$$+(1+\lambda)B_{-1}\left(-\Delta\alpha(\alpha-1)K^{\alpha-2}Q^{B^2}\right)$$

For this whole expression to be negative, it's enough to show that the sum of these two terms are negative, as the second and third terms of the previous expression are both negative:

$$\left[\frac{N(1+\lambda)^2}{\Theta(M/N)}\left(\beta\Lambda\alpha(\alpha-1)K^{\alpha-2}\right)\right] + (1+\lambda)B_{-1}\left(-\Delta\alpha(\alpha-1)K^{\alpha-2}Q^{B^2}\right) < 0$$
$$= \frac{(1+\lambda)\alpha(\alpha-1)K^{\alpha-2}Q^B}{\Theta(M/N)}\left[\frac{N\beta\Lambda(1+\lambda)}{Q^B} - \Theta(M/N)\Delta Q^B B_{-1}\right]$$

As the term outside the parenthesis is negative, we need the one inside to be positive, for the whole expression to be negative:

$$\frac{N\beta\Lambda(1+\lambda)}{Q^B} - \Theta(M/N)\Delta Q^B B_{-1}$$
$$= N\beta\Lambda(1+\lambda)R^B - \Theta(M/N)\Delta Q^B B_{-1}$$
$$\geq N\beta\Lambda(1+\lambda)R^D - \Theta(M/N)\Delta Q^B B_{-1}$$
$$= (1+\lambda)N - \Theta(M/N)\Delta Q^B B_{-1}$$

Now we use that $N = X + Q^B B_{-1}$

$$= (1+\lambda)X + Q^B B_{-1}\big(1+\lambda - \Theta(M/N)\Delta\big) > 0$$

since $1+\lambda > \Theta(M/N)\Delta$

So we proved $\frac{dK}{dM} > 0$

$$\frac{dQ^B}{dM} = \underbrace{-\Delta\alpha(\alpha-1)K^{\alpha-2}Q^{B^2}}_{(+)}\underbrace{\frac{dK}{dM}}_{(+)}$$

The reserves convenience yield, using the banks optimality conditions of bonds and reserves is:

$$(R^D - R^M) = \underbrace{(R^D - R^B)}_{(-)}\underbrace{(S + \Delta^L Q^B B)}_{(+)}\underbrace{\frac{\Theta'(M/N)}{\Delta^L\Theta(M/N)}}_{(-)}$$

which is positive.

9.17 Government Bonds and The Central Bank Balance Sheet Process

We know the price of such a bond with a coupon decaying at rate ρ and starting with $\rho_0 = 1$ next period, and using the discount factor $\beta = \frac{1}{1+r}$ is given by

$$P = \rho^{-1}\left[(\beta\rho)^1 + (\beta\rho)^2 + \ldots\right] = \rho^{-1}\frac{\beta\rho}{1-\beta\rho} = \rho^{-1}\frac{\rho(1+r)}{1-\rho(1+r)} = \frac{\rho^{-1}\rho(1+r)}{(1+r)-\rho(1+r)} = \frac{1}{(1+r)-\rho}.$$

So,

$$\frac{\partial P}{\partial r} = -\frac{1}{\left[(1+r)-\rho\right]^2} = -\frac{1}{(1+r-\rho)}P$$

and therefore the duration is

$$D = -\frac{\frac{\partial P}{\partial r}}{P} = \frac{1}{(1+r-\rho)}.$$

The Macaulay duration D_M, which is related via $D = D_M \cdot (1+r)$.

Thus:

$$D_M = D \cdot (1+r) = \frac{1+r}{(1+r-\rho)} = \frac{1}{1-\frac{\rho}{1+r}} = \frac{1}{1-\rho\beta} = (1-\rho\beta)^{-1}.$$

Using $\kappa = \rho\beta$ the duration is $\frac{1}{1-\kappa}$.

The purchases of bonds by the central bank can be expressed as:

$$\begin{aligned}\Psi_t &= \gamma^b Q_t^B B_{t-1}^{CB} - Q_{t-1}^B B_{t-1}^{CB} \\ &= -B_{t-1}^{CB} + \gamma^b Q_t^B B_{t-1}^{CB} - Q_{t-1}^B B_{t-1}^{CB} + B_{t-1}^{CB}\end{aligned}$$

Using the YTM definition: $R_t^L = \frac{1}{Q_t^B} + \gamma^b$:

$$\begin{aligned}\Psi_t &= -B_{t-1}^{CB} + \gamma^b Q_t^B B_{t-1}^{CB} - Q_{t-1}^B B_{t-1}^{CB} + B_{t-1}^{CB} \\ &= (Q_t^B R_t^L - Q_{t-1}^B) B_{t-1}^{CB} \\ &= (\frac{Q_t^B}{Q_{t-1}^B} R_t^L - 1) Q_{t-1}^B B_{t-1}^{CB}\end{aligned}$$

So the passive change in the portfolio is:

$$\Psi_t = \Bigg(\underbrace{(-(Q^B_{t-1}))^{-1}}_{\text{Runoff}} + \underbrace{\frac{Q^B_t}{Q^B_{t-1}} R^L_t - 1}_{\text{Revaluation}} \Bigg) Q^B_{t-1} B^{CB}_{t-1}$$

So the total reinvestment is the passive change plus new purchases/announcements:

$$TR_t = \rho \Psi_t + \sum_{j \geq 0}^{T} \epsilon^{CB}_{t|t-j}$$

We can express the CB balance sheet evolution as:

$$\begin{aligned} Q^B_t B^{CB}_t &= Q^{CB}_{t-1} B^{CB}_{t-1} + \Psi_t + TR_t \\ &= \Big[1 + (1-\rho)\big(-(Q^B_{t-1})^{-1} + \Pi_t R^L_t - 1 \big) \Big] Q^{CB}_{t-1} B^{CB}_{t-1} + \sum_{j \geq 0}^{T} \epsilon^{CB}_{t|t-j} \end{aligned}$$

In the main text, the steady-state size of the balance sheet is not zero.

9.18 *Optimal Policy Projection: Two-Instrument Problem*

We start with the commitment scenario: policy shocks involve shocks to the interest rate, denoted as $\epsilon_{R^M,t+k|t}$, and shocks to asset holdings, $\epsilon_{B^{CB},t+k|t}$, for $0 \leq k \leq T$. The policy shock vector as

$$\epsilon_t \equiv \begin{pmatrix} \epsilon_t^{R^M} \\ \epsilon_t^{CB} \end{pmatrix}, \tag{29}$$

with $\epsilon_t^j \equiv (\epsilon_{t|t}^j, \epsilon_{t+1|t}^j, ..., \epsilon_{t+T|t}^j)'$, $j \in \{R^M, B^{CB}\}$. The policy shock vector ϵ_t stacks two instrument-specific shock vectors below each other, doubling the size of ϵ_t to $2(T+1)$ elements.

The use of two instruments also requires two instrument-specific IRF vectors. Let the vector $d_j^{x,k} \equiv (d_{0,j}^{x,k}, d_{1,j}^{x,k}, ...d_{T,j}^{x,k})'$, $j \in \{R^M, B^{CB}\}$ contain the impulse response coefficients for variable $x \in \{\Pi, R_t^M, B_t^{CB}, Y\}$ and the instrument-specific policy shock $\epsilon_{j,t+k|t}$, $j \in \{R^M, B^{CB}\}$. For the shock vector ϵ_t, the value of variable x in period $t+s$ can then be written as:

$$x_{t+s} = \sum_{j \in \{R^M, B^{CB}\}} \sum_{0 \leq k \leq T} d_j^{x,k} \epsilon_{j,t+k|t}, \tag{30}$$

To write the entire path for variable x, given by $X_t = (x_t, x_{t+1}, x_{t+2}, \ldots, x_{t+T})'$, in a convenient manner, first define the instrument-specific coefficient matrix

$$D_x^j \equiv \begin{pmatrix} d_{0,j}^{x,0} & d_{0,j}^{x,1} & \cdots & d_{0,j}^{x,T} \\ d_{1,j}^{x,0} & d_{1,j}^{x,1} & \cdots & d_{1,j}^{x,T} \\ \vdots & \vdots & \ddots & \vdots \\ d_{T,j}^{x,0} & d_{T,j}^{x,1} & \cdots & d_{T,j}^{x,T} \end{pmatrix}. \tag{31}$$

Now, define the $(T+1) \times 2(T+1)$-dimensional overall coefficient matrix D_x by stacking the instrument-specific coefficient matrices next to each other, i.e., $D_x \equiv [D_x^{R^M}, D_x^{B^{CB}}]$. As in the one-instrument case, the time path X_t can then be computed as $X_t = D_x \epsilon_t$. However, the individual elements of X_t are now given by equation (24).

The next step is to define the outcome vector

$$Z_t \equiv \begin{pmatrix} \Pi_t \\ Y_t \\ R_t^M - R_{t-1}^M \\ B_t^{CB} \\ B_t^{CB} - B_{t-1}^{CB} \end{pmatrix}, \tag{32}$$

the baseline vector

$$B_t \equiv \begin{pmatrix} B_t^{\pi} \\ B_t^{y} \\ B_t^{\Delta R^M} \\ B_t^{B^{CB}} \\ B_t^{\Delta B^{CB}} \end{pmatrix}, \tag{33}$$

and the coefficient matrix

$$D \equiv \begin{pmatrix} D^{\pi} \\ D^{y} \\ D^{\Delta R^M} \\ D^{B^{CB}} \\ D^{\Delta B^{CB}} \end{pmatrix}. \tag{34}$$

The policy problem is:

$$\min_{\epsilon_t} \left\{ \frac{1}{2} Z_t' W Z_t \right\} \quad \text{s.t.} \quad Z_t = B_t + D\epsilon_t. \tag{35}$$

However, the matrix W is now given as

$$W = \begin{pmatrix} W_\beta & 0 & 0 & 0 & 0 \\ 0 & \lambda W_\beta & 0 & 0 & 0 \\ 0 & 0 & w_{R^M} W_\beta & 0 & 0 \\ 0 & 0 & 0 & w_{B^{CB}} W_\beta & 0 \\ 0 & 0 & 0 & 0 & w_{\Delta B^{CB}} W_\beta \end{pmatrix}. \tag{36}$$

Unconstrained scenario solution:

$$\epsilon_t^* = -(D'WD)^{-1}(B_t'WD)', \tag{37}$$

For the constrained case:

$$\begin{gathered} \min_{\epsilon_t} \frac{1}{2} Z_t' W Z_t \\ \text{s.t.} \quad Z_t = \underbrace{B_t}_{Baseline} + \underbrace{D\epsilon_t}_{IR} \\ B_t^{R^M} + D^{R^M} \epsilon \geq 1 \quad \text{(ZLB)} \\ -D^{B^{CB}} \epsilon_t \leq B_t^{B^{CB}} \quad \text{(BS Lower Bound)} \\ B_t^{B^{CB}} + D^{B^{CB}} \epsilon \leq \bar{B}^{CB} \quad \text{(BS Upper Bound)} \end{gathered}$$

Now we proceed with the algorithm for the constrained time-consistent problem. We guess a vector of policy announcements for the policy interest rate and the QE and subsequent QT strategies. For each period in the projection we solve for the optimal unanticipated shock: if the latter is 0, the planner will not deviate. If not, update the guess.

1. Set the initial guess $\epsilon_t^{*,0}$

2. Calculate $Z^k = B + D\epsilon_t^{*,0}$ so the projection from j to T is $Z^k_{j:T}$

3. For each j solve the minimization problem as stated in the formulation of the commitment case. As it's a constrained case, the solution does not have closed form.

4. If the solution (optimal size of the shock) is lower than the imposed threshold, end. Else, use an update and return to step 2.

9.19 The Role of Expectations, Credibility and Finite Planning

To mitigate the forward guidance, we use the method of inattention proposed by de Groot and Mazelis (2020) where a fraction χ^H of agents are attentive and work as a standard rational expectation model, where $m_t = \mathbb{E}_t\, m_{t+1}$, and the remaining agents are innatentive and set $m_t = 0$. Iterating forward and aggregating across agents: $m_t = \chi^{H,t}\, \mathbb{E}_t\, m_{t+h}$. So we proceed to scale the impact matrix that in our setting is $D\epsilon^t$ by a matrix T containing the fraction of attentive agents.

In the case of rational expectations each element of T is 1.

For credibility, a fraction χ^H of agents incorporate conventional and unconventional shocks H periods ahead in the future into their expectations. Finally, under finite planning agents dismiss announcement that are more than N periods ahead.

For a full treatment on how to solve in a general matrix form these type of models we refer the reader to de Groot et al. (2021) and de Groot and Mazelis (2020).

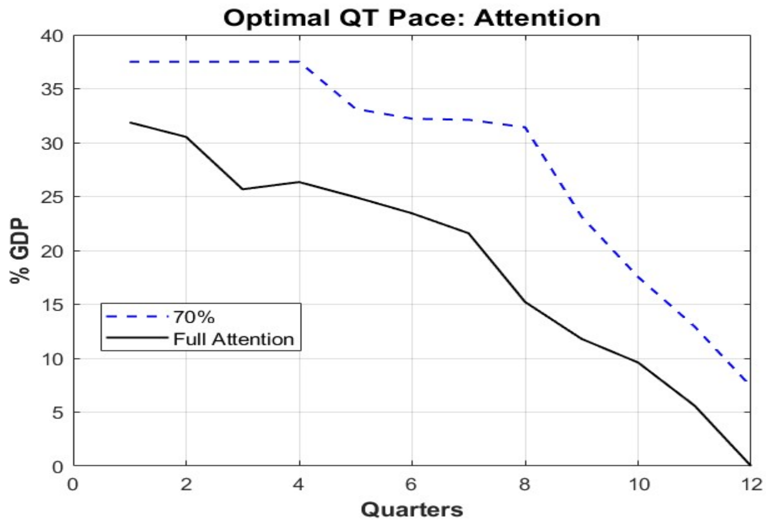


Figure 39: Optimal QT Pace: The role of attention

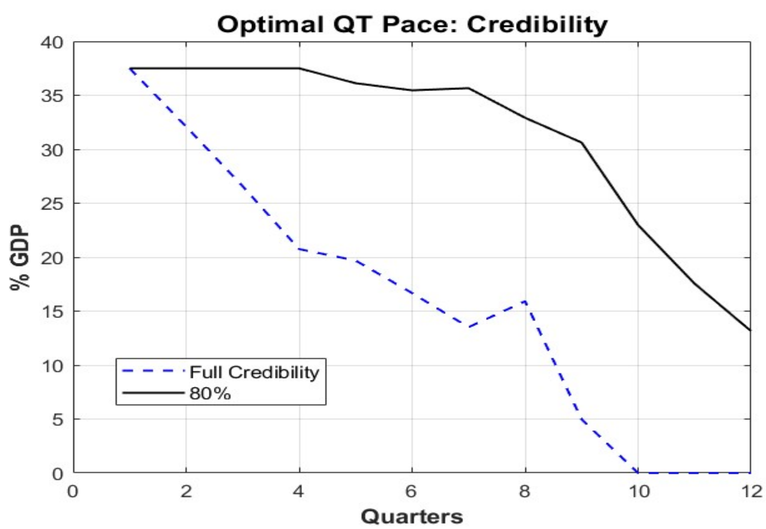


Figure 40: Optimal QT Pace: The role of Credibility

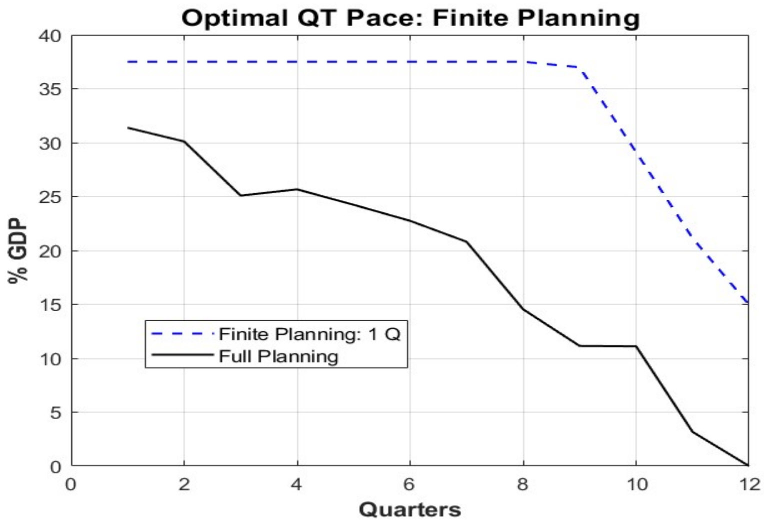


Figure 41: Optimal QT Pace: The role of Finite Planning

9.20 The Role of Household Heterogeneity

We extend the analysis to study the role of household heterogeneity adding Hand-to-Mouth agents.

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^j \left[\frac{(C_{t+j}^{Htm})^{1-\gamma}}{1-\gamma} - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi, Htm} \right]$$

$$st.$$

$$C_t^{Htm} = W_t L_t^{Htm} + T_t^{Htm}$$

The fiscal rule is set to model fiscal stimulus:

$$T_t^{HTM} = T^{HTM} + \theta^G (Y - Y_t) + \epsilon_t^F$$

We will explore the credit evolution under QT transition when hand to mouth agents is 12%.

$$Q_t^K K_t = \frac{\nu_t^d N_t}{\Theta\left(\frac{M_t}{N_t}\right) - \nu_t^k (1 + \Psi_t)} - \Delta^L Q_t^B \left(B^G - B_t^H - B_t^{CB}\right)$$

We'll index hand-to-mouth agents with HtM and the ricardian ones with U.

Replacing the household optimal demand for government bonds and the market clearing conditions $B_t^H = (1 - \omega^{HtM}) B_t^{H,U}$:

$$Q_t^K K_t = \frac{\nu_t^d N_t}{\Theta\left(\frac{M_t}{N_t}\right) - \nu_t^k (1 + \Psi_t)} - \Delta^L Q_t^B \left(B^G - (1 - \omega^{HtM}) \left(\overline{B}^H + \frac{\mathbb{E}_t \Lambda_{t+1} (R_{t+1}^B - R_t^d)}{\kappa^H} \right) - B_t^{CB} \right)$$

As the hand-to-mouth agents share increase, the household block absorbs less government bonds, so banks experience higher capital losses and the rebalancing channel is higher, leading to a higher credit crunch.

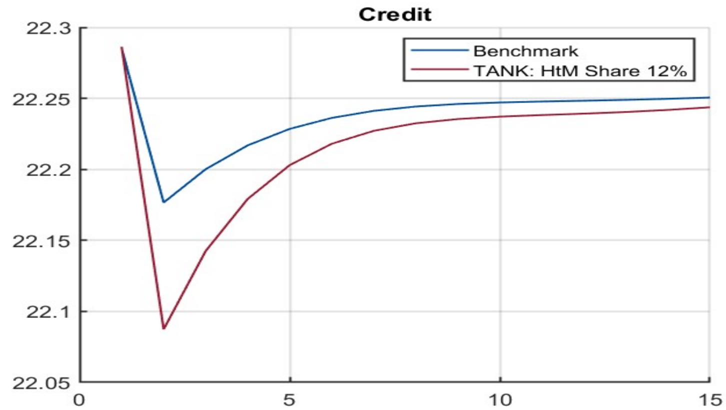


Figure 42: Evolution of credit. The blue line is the benchmark economy. The red line shows the evolution of credit when there are hand-to-mouth agents.

As consumption will decrease more on impact with QT, inflation will also decrease more, unless there's enough fiscal stimulus. (higher θ^G). With HtM agents, the decrease in deposits is lower.

9.21 The Planner's Loss Function: a Hawkish Planner and AIT

We decrease the weights of the output gap to 0.075. In this scenario QT is more aggressive under discretion (3.04%) and commitment (4.03%) for the first three years, aside from higher average short-term interest rates.

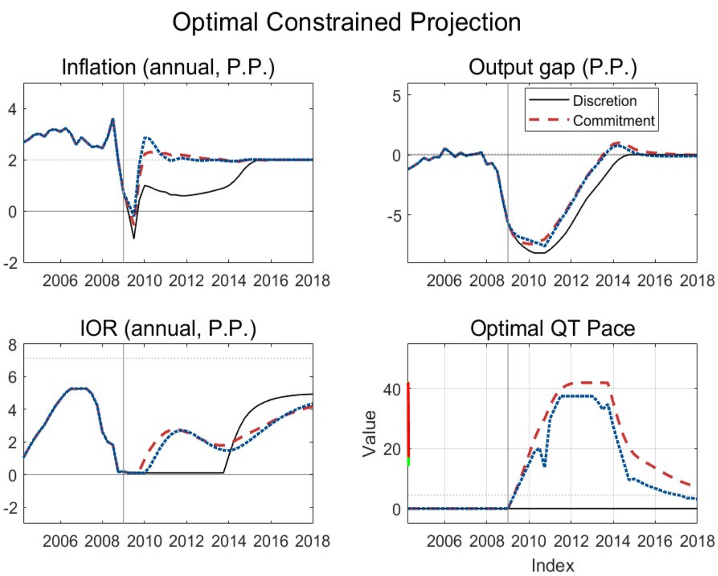


Figure 43: QT Optimal Paces: Balance Sheet as Percent of GDP

We compare the benchmark scenario with an Average Inflation Targeting. The latter delivers a better performance for the output gap, and QT

9.22 *An extension with Non-Banks FI and ONRRP (TBC)*

TBD

- Banks invest in loans and reserves. They take Deposits from HHs
- Dealers: invest in bonds with repos
- HHs
- MMFs invest in ONRRP or repos with shares from HHs
- Two channels: reserve demand by banks and the repo. The second one: QT increases the demand for financing in the repo market and the demand for liquidity by non banks. QT non banks lend more in repo market.
- Decline in the CB BS increases the amount of securities needed to be financed in the repo market, so markets increase. Non Banks reduce lending at ON RRP and lend more in repo. MMFs inflwo of deposits.

9.22.1 *Households*

Now households can invest in shares at the mutual funds. These are subject to portfolio costs. Their budget constraint is now:

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(C_{t+j}^U)^{1-\gamma}}{1-\gamma} - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi,U} \right]$$

$$st.$$

$$C_t^U + D_t + T_t^U + A_t + \Phi_t^A = W_t L_t^U + R_t^D D_{t-1} + A_{t-1} R_t^A + \sum_{B,F} \Pi_t^U$$

9.22.2 *Dealers*

$$Z_t = Q_t^B B_t^D$$

$$\Pi_t^D = R_{t+1}^B Q_t^B B_t^D - R_t^Z Z_t - \Phi_t(Q_t^B B_t^D)$$

$$\Phi_t(Q_t^B B_t^D) = \frac{1}{2}\frac{\kappa^D(B_t^D - \bar{B^D})^2}{B_t^D}Q_t^B B_t^D$$

Optimality

$$R_{t+1}^B - R_t^Z = \Phi_t'(Q_t^B B_t^D)$$
$$B_t^D = \bar{B^D} + \frac{R_{t+1}^B - R_t^Z}{\kappa^D}$$

9.22.3 Mutual Funds

Balance Sheet:

$$Z_t + O_t = A_t$$

The profits of the mutual funds is:

$$\Pi_t^{MF} = \left(R_t^Z - R_t^A\right)Z_t + \left(R_t^O - R_t^A\right)O_t - \Phi(A_t, Z_t, O_t)$$

$$\Phi(Q_t^B B_t^F, O_t) = -\epsilon^D O_t + \frac{\epsilon^Z}{2}(Z_t - Z_{t-1})^2$$

The optimality conditions are given by:

$$R_t^Z - R_t^A = \Phi_t^Z(A_t, Z_t, O_t)$$

$$R_t^O - R_t^A = \Phi_t^O(A_t, Z_t, O_t)$$

9.22.4 Only MMFs

$$Q_t^B B_t^{MF} + O_t = A_t$$

$$\Pi_t^{MF} = (R_t^O - R_t^A)O_t + (R_{t+1}^B - R_t^A)Q_t^B B_t^{MF} - \Phi(Q_t^B B_t^{MF}, O_t)$$

(a) Mutual Funds

Assets	Liabilities
ONRRP	HH shares
Repo	

(b) Banks

Assets	Liabilities
Loans	Deposits
Government Bonds	Net-Worth
Reserves	

(c) Dealers

Assets	Liabilities
Government Bonds	Repo

(d) Central Bank

Assets	Liabilities
Government Bonds	Reserves
	ONRRP

Figure 44: Balance Sheets

Optimality:

$$R_t^O - R_t^A = \Phi^O(Q_t^B B_t^{MF}, O_t)$$
$$R_{t+1}^B - R_t^A = \Phi_t^A(Q_t^B B_t^{MF}, O_t)$$

TBD